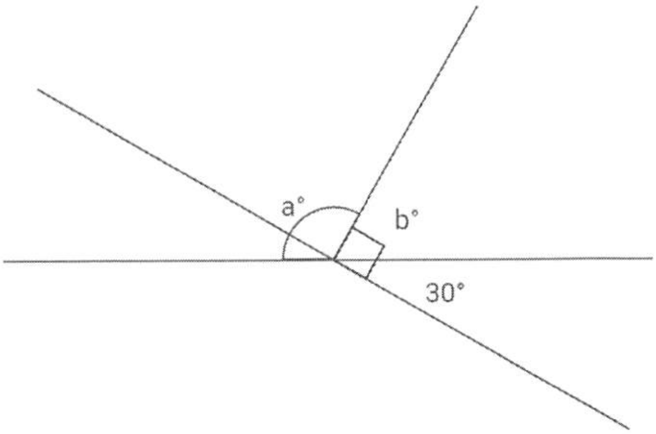
Homework Helpers

Grade 7
Module 6

G7-M6-Lesson 1: Complementary and Supplementary Angles

1. Two lines meet at the endpoint of a ray. Set up and solve the appropriate equations to determine a and b.



$$90 + b + 30 = 180$$
$$120 + b = 180$$
$$120 - 120 + b = 180 - 120$$
$$b = 60$$

Angles on a line have a sum of 180°.

I use the additive inverse to solve for the variable.

I already determined b is equal to 60.

$$a + b = 180$$
$$a + 60 = 180$$
$$a + 60 - 60 = 180 - 60$$
$$a = 120$$

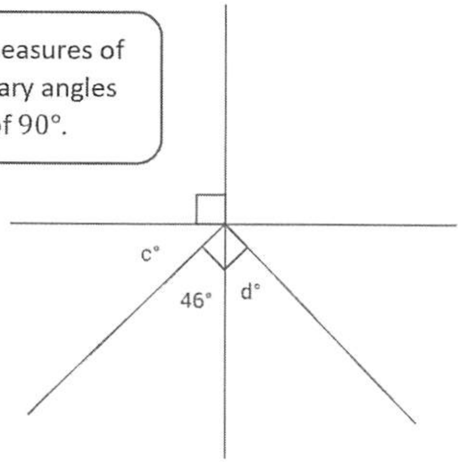
The angles identified by $a°$ and $b°$ are also angles on a line, so their measures have a sum of 180°.

Therefore, the angles identified by $a°$ and $b°$ have measures of 120° and 60°, respectively.

2. Two lines meet at the common endpoint of two rays. Set up and solve the appropriate equations to determine c and d.

$$46 + c = 90$$
$$46 - 46 + c = 90 - 46$$
$$c = 44$$

I know the measures of complementary angles have a sum of $90°$.

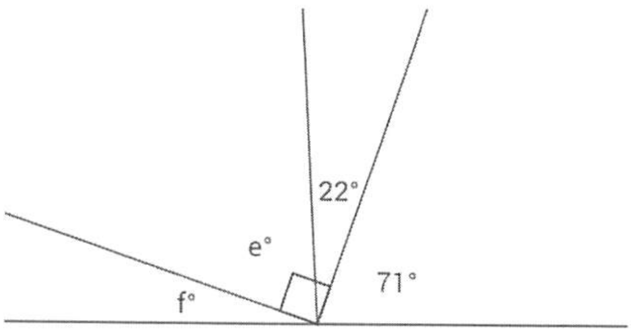


$$46 + d = 90$$
$$46 - 46 + d = 90 - 46$$
$$d = 44$$

Therefore, the angles identified by $c°$ and $d°$ both have measures of $44°$.

I could also recognize that c and d have equal values because the angles they are identified with are both complements to the same angle.

3. Set up and solve appropriate equations for e and f.



$$22 + e = 90$$
$$22 - 22 + e = 90 - 22$$
$$e = 68$$

$$f + e + 22 + 71 = 180$$
$$f + 68 + 22 + 71 = 180$$
$$f + 161 = 180$$
$$f + 161 - 161 = 180 - 161$$
$$f = 19$$

I already determined that e has a value of 68.

Therefore, the angles identified by $e°$ and $f°$ have measures of $68°$ and $19°$, respectively.

4. The measurement of the supplement of an angle is $30°$ more than double the measurement of the angle. Find the measurements of the angle and its supplement.

Let $x°$ represent the measurement of the angle.

$$x + (2x + 30) = 180$$
$$3x + 30 = 180$$
$$3x + 30 - 30 = 180 - 30$$
$$3x = 150$$
$$\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)150$$
$$x = 50$$

I know the measures of supplementary angles have a sum of $180°$.

I use my knowledge of solving equations to determine the value of x.

Let $(2x + 30)°$ represent the measurement of the supplement of the angle.

$$2x + 30$$
$$= 2(50) + 30$$
$$= 100 + 30$$
$$= 130$$

To find the measure of the supplement, I can either subtract the measure of the angle from $180°$, or I can substitute the value of x into the expression $2x + 30$.

The angle measures $50°$, and its supplement measures $130°$.

5. The measurement of the complement of an angle exceeds the measurement of the angle by 50%. Find the measurements of the angle and its complement.

Let $x°$ represent the measurement of the angle and let $(x + 0.5x)°$ represent the measurement of its complement.

$$x + (x + 0.5x) = 90$$
$$2.5x = 90$$
$$\left(\frac{1}{2.5}\right)2.5x = \left(\frac{1}{2.5}\right)90$$
$$x = 36$$

The measure of the complement of the angle is the sum of the measure of the angle plus another 50% of its measure.

$$90 - 36 = 54$$

We could also substitute 36 into the expression $x + 0.50x$ to determine the measure of the complement.

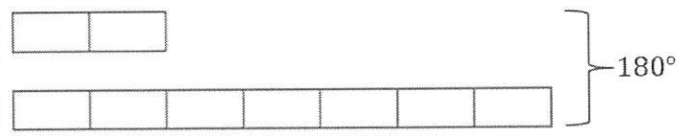
The angle measures $36°$ and its complement measures $54°$.

6. The ratio of the measurement of an angle to the measurement of its supplement is $2:7$. Find the measurements of the angle and its supplement.

Let $2x°$ represent the measurement of the angle and let $7x°$ represent the measurement of its supplement.

$$2x + 7x = 180$$
$$9x = 180$$
$$\left(\frac{1}{9}\right)9x = \left(\frac{1}{9}\right)180$$
$$x = 20$$

The ratio could also be described with a tape diagram.



I have found x, but this does not yet answer the question.

Angle: $2(20)° = 40°$ *Supplement:* $7(20)° = 140°$

Therefore, the angle measures $40°$ and its supplement measures $140°$.

G7-M6-Lesson 2: Solving for Unknown Angles Using Equations

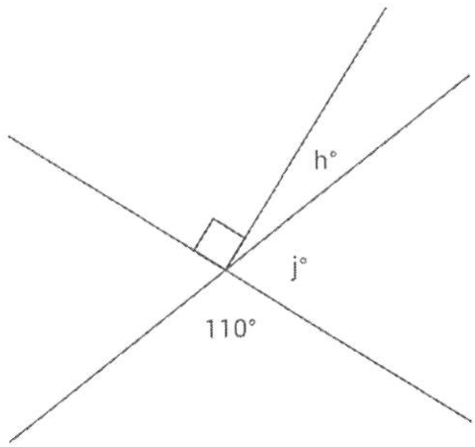
1. Two lines meet at the endpoint of a ray. Set up and solve an equation to find the values of h and j.

I know that vertical angles have the same angle measure.

$$110 = 90 + h$$
$$110 - 90 = 90 - 90 + h$$
$$20 = h$$

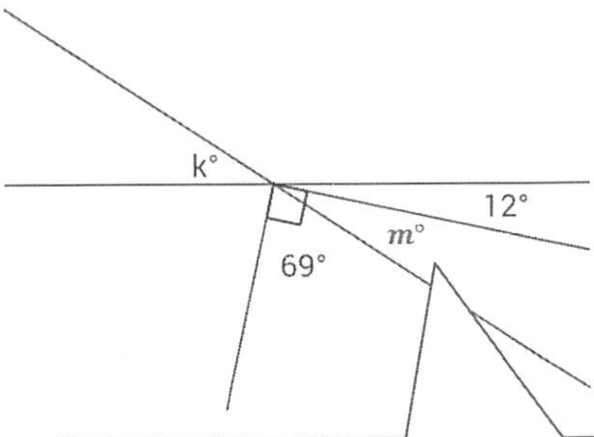
I know that the measures of angles on a line have a sum of 180°.

$$110 + j = 180$$
$$110 - 110 + j = 180 - 110$$
$$j = 70$$



The value of h is 20 and the value of j is 70.

2. Two lines meet at the vertex of an angle formed by two rays. Set up and solve an equation to find the value of k.



I need to determine the measure of this unknown angle in order to determine the value of k. I'll let this angle have a measure of $m°$.

I know that the measures of complementary angles have a sum of 90°.

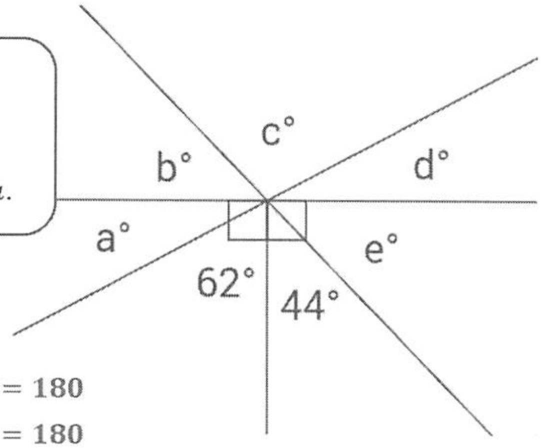
$$69 + m = 90$$
$$69 - 69 + m = 90 - 69$$
$$m = 21$$

I know the measures of two adjacent angles that form a vertical angle to the angle $k°$.

$$21 + 12 = k$$
$$33 = k$$

The value of k is 33.

3. Three lines meet at the endpoint of a ray. Set up and solve an equation to find the value of each variable in the diagram.



$$62 + 44 = c$$
$$106 = c$$

I can use my knowledge of vertical angles and complementary angles to determine the values of c and a.

$$a + 62 = 90$$
$$a + 62 - 62 = 90 - 62$$
$$a = 28$$

$$b + a + 62 + 44 = 180$$
$$b + 28 + 62 + 44 = 180$$
$$b + 134 = 180$$
$$b + 134 - 134 = 180 - 134$$
$$b = 46$$

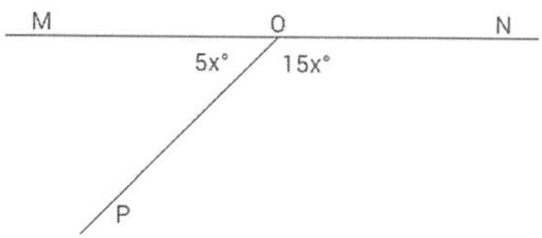
Now that I know the value of a, I can calculate the value of b because I know that the measures of angles on a line have a sum of 180°.

Now, I can use my knowledge of vertical angles to determine the values of e and d.

$$b = e$$
$$46 = e$$

$$a = d$$
$$28 = d$$

4. Set up and solve an equation to find the value of x. Find the measurements of $\angle MOP$ and $\angle NOP$.



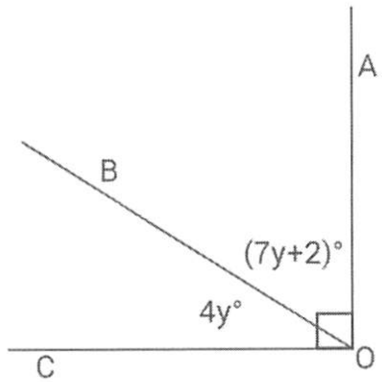
$$5x + 15x = 180$$
$$20x = 180$$
$$\left(\frac{1}{20}\right)20x = \left(\frac{1}{20}\right)180$$
$$x = 9$$

I can use the value of x to determine the measure of the two unknown angles.

$$\angle MOP = 5x^\circ = 5(9)^\circ = 45^\circ$$
$$\angle NOP = 15x^\circ = 15(9)^\circ = 135^\circ$$

I can check my answers by making sure the measures of the two angles have a sum of 180°.

5. Set up and solve an equation to find the value of y. Find the measurements of $\angle AOB$ and $\angle BOC$.



I need to collect like terms before solving the equation.

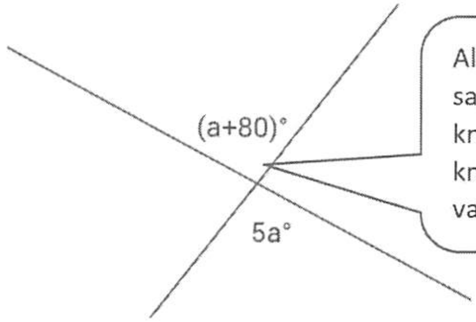
$$4y + 7y + 2 = 90$$
$$11y + 2 = 90$$
$$11y + 2 - 2 = 90 - 2$$
$$11y = 88$$
$$\left(\frac{1}{11}\right)11y = \left(\frac{1}{11}\right)88$$
$$y = 8$$

$$\angle BOC = 4y^\circ = 4(8)^\circ = 32^\circ$$
$$\angle AOB = (7y + 2)^\circ = (7(8) + 2)^\circ = 58^\circ$$

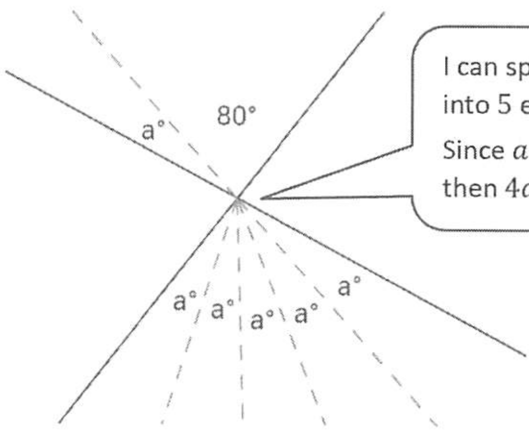
I can check my answers by making sure the measures of the two angles have a sum of 90°.

G7-M6-Lesson 3: Solving for Unknown Angles Using Equations

1. Two lines meet at a point. Find the measurement of a vertical angle. Is your answer reasonable? Explain how you know.



Although I know vertical angles have the same measure, I cannot use that knowledge immediately because I do not know how to solve an equation with variables on both sides of the equal sign.



I can split each angle into 5 equal pieces. Since $a + 4a = a + 80$, then $4a = 80$.

$$4a = 80$$
$$\left(\frac{1}{4}\right)4a = \left(\frac{1}{4}\right)(80)$$
$$a = 20$$

I can also solve the problem this way:

$$5a = a + 80$$
$$5a - a = a - a + 80$$
$$4a = 80$$
$$\left(\frac{1}{4}\right)4a = \left(\frac{1}{4}\right)(80)$$
$$a = 20$$

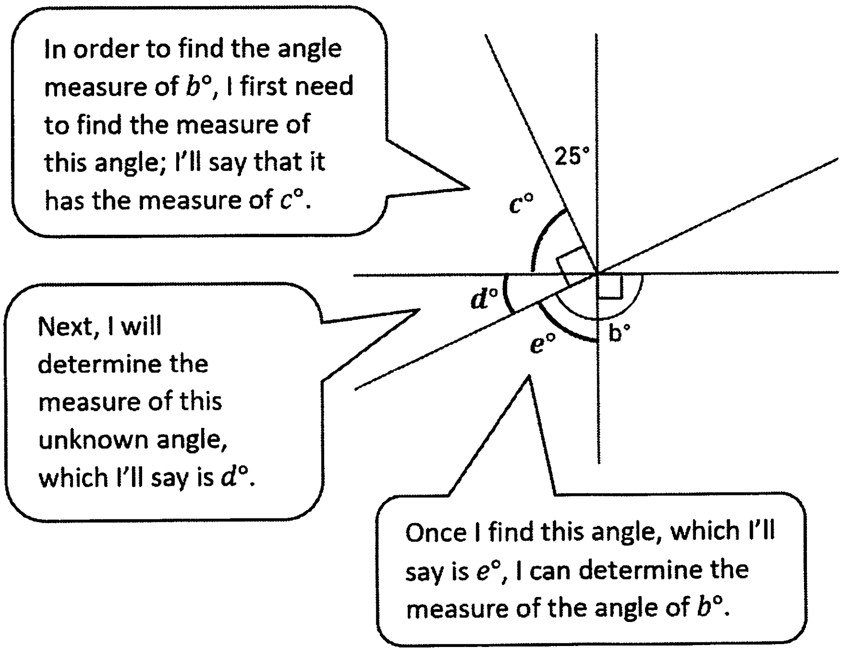
$a + 80 = 20 + 80 = 100$

$5a = 5(20) = 100$

Therefore, each vertical angle has a measure of $100°$.

My answer is reasonable because the vertical angle looks to be close to the measure of a right angle.

2. Three lines meet at the endpoint of a ray. Set up and solve an equation to find the value of b.



$$c + 25 = 90$$
$$c + 25 - 25 = 90 - 25$$
$$c = 65$$

$$c + d = 90$$
$$65 + d = 90$$
$$65 - 65 + d = 90 - 65$$
$$d = 25$$

$$d + e = 90$$
$$25 + e = 90$$
$$25 - 25 + e = 90 - 25$$
$$e = 65$$

Looking at the diagram, I see that the angle of measure $b°$ consists of a right angle and the angle of measure $e°$.

$90 + 65 = 155$

Therefore, the value of b is 155.

3. Four angles meet at a point. The second angle measures $10°$ more than the first angle, the third angle measures $15°$ more than the second angle, and the fourth angle measures $20°$ more than the third angle. Find the measurements of all four angles.

I know that the measures of angles that meet at a point have a sum of $360°$.

Let x represent the value of the first angle measurement.

$$(x) + (x + 10) + (x + 10 + 15) + (x + 10 + 15 + 20) = 360$$
$$4x + 80 = 360$$
$$4x + 80 - 80 = 360 - 80$$
$$4x = 280$$
$$\left(\frac{1}{4}\right)4x = \left(\frac{1}{4}\right)280$$
$$x = 70$$

Each set of parentheses represents the measure of each of the angles.

The following are the measures of each of the angles:

Angle 1: $70°$

Angle 2: $(70)° + 10° = 80°$

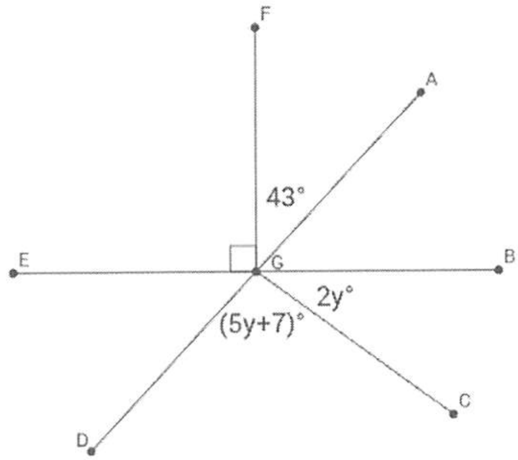
Angle 3: $(70)° + 10° + 15° = 95°$

Angle 3: $(70)° + 10° + 15° + 20° = 115°$

To check my answers, I could add the measures of the four angles together to determine if they have a sum of $360°$.

G7-M6-Lesson 4: Solving for Unknown Angles Using Equations

1. $\overleftrightarrow{BE}$ and $\overleftrightarrow{AD}$ meet at G. Set up and solve an equation to find the value of y. Find the measurements of $\angle BGC$ and $\angle CGD$.



$$43 + \angle AGB = 90$$
$$43 - 43 + \angle AGB = 90 - 43$$
$$\angle AGB = 47$$

I need to determine the measurement of $\angle AGB$ before finding the measurements of $\angle BGC$ and $\angle CGD$.

$$\angle AGB + 2y + 5y + 7 = 180$$
$$47 + 2y + 5y + 7 = 180$$
$$54 + 7y = 180$$
$$54 - 54 + 7y = 180 - 54$$
$$7y = 126$$
$$y = 18$$

Now that I know the value of y, I can calculate the measurements of $\angle BGC$ and $\angle CGD$.

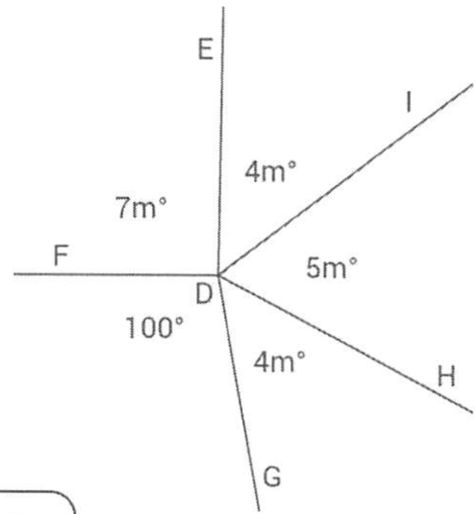
The measurement of $\angle BGC$: $2(18)^\circ = 36^\circ$

The measurement of $\angle CGD$: $(5(18) + 7)^\circ = (90 + 7)^\circ = 97^\circ$

2. Five rays meet at a point. Set up and solve an equation to find the value of m. Find the measurements of $\angle EDF$ and $\angle HDG$.

$$7m + 4m + 5m + 4m + 100 = 360$$
$$20m + 100 = 360$$
$$20m + 100 - 100 = 360 - 100$$
$$20m = 260$$
$$\left(\frac{1}{20}\right) 20m = \left(\frac{1}{20}\right) 260$$
$$m = 13$$

I can use the value of m to determine the measures of the unknown angles.



$\angle EDF = 7m° = 7(13)° = 91°$

$\angle HDG = 4m° = 4(13)° = 52°$

I only need to determine the measures of the two angles presented in the question.

3. Three adjacent angles form a line. The measurement of each angle is one of three consecutive, positive whole numbers. Determine the measurements of all three angles.

Consecutive numbers are ones that directly follow each other. For example, 2, 3, 4.

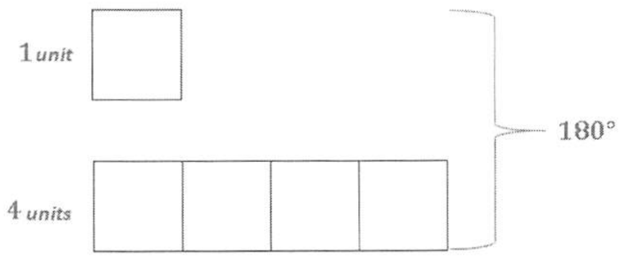
Let $x°$ represent the smallest angle measure.

Since $x°$ represents the measure of the smallest angle, then the measure of the second angle is 1° larger than $x°$, and the measure of the third angle is 2° larger than $x°$.

$$x + (x + 1) + (x + 2) = 180$$
$$3x + 3 = 180$$
$$3x + 3 - 3 = 180 - 3$$
$$3x = 177$$
$$\left(\frac{1}{3}\right) 3x = \left(\frac{1}{3}\right) 177$$
$$x = 59$$

The three angles measure 59°, 60°, and 61° because I determined the smallest angle measures 59°, and the measures of the other two angles are consecutive numbers.

4. The ratio of measurement of an angle to the measurement of its supplement is $1:4$.



$5\ \textit{units} = 180°$

$1\ \textit{unit} = 36°$

$4\ \textit{units} = 144°$

To find the value of one unit, I divide $180°$ by 5 because I know 5 units has a value of $180°$.

The measure of the angle that satisfies these criteria is $36°$.

5. The sum of four times the measurement of the complement of an angle and the measurement of the supplement of that angle is $240°$. What is the measurement of the angle?

Let $a°$ represent the measurement of the angle.

I use my knowledge of complements and supplements to write an equation.

$$4(90 - a) + (180 - a) = 240$$
$$360 - 4a + 180 - a = 240$$
$$540 - 5a = 240$$
$$540 - 540 - 5a = 240 - 540$$
$$-5a = -300$$
$$\left(-\frac{1}{5}\right)(-5a) = \left(-\frac{1}{5}\right)(-300)$$
$$a = 60$$

I need to be careful when collecting like terms. I remember that subtracting is the same as adding the opposite, so I could rewrite the equation as follows: $360 + (-4a) + 180 + (-a) = 240$.

The measurement of the angle is $60°$.

G7-M6-Lesson 5: Identical Triangles

1. Given the following triangles' correspondences, use double arrows to show the correspondence between vertices, angles, and sides.

The order in which the vertices are listed for each triangle is important. The correspondence of vertices, angles, and sides are determined by the order of the label.

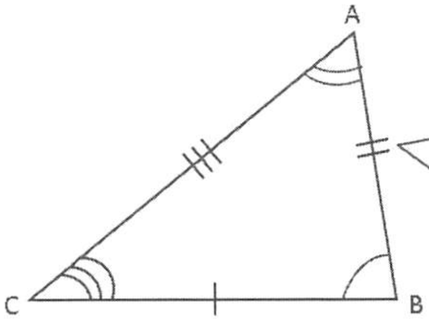
The same correspondence rule for vertices is true for angle correspondence.

Triangle Correspondence	$\triangle MNP \leftrightarrow \triangle XYZ$
Correspondence of Vertices	$M \longleftrightarrow X$ $N \longleftrightarrow Y$ $P \longleftrightarrow Z$
Correspondence of Angles	$\angle M \longleftrightarrow \angle X$ $\angle N \longleftrightarrow \angle Y$ $\angle P \longleftrightarrow \angle Z$
Correspondence of Sides	$\overline{MN} \longleftrightarrow \overline{XY}$ $\overline{NP} \longleftrightarrow \overline{YZ}$ $\overline{MP} \longleftrightarrow \overline{XZ}$

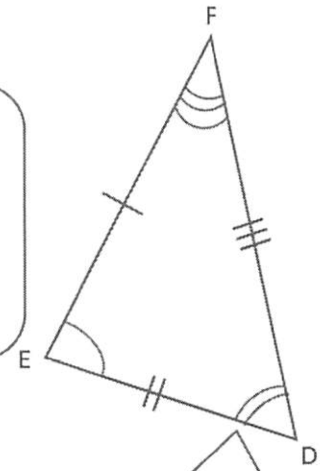
The first vertex labeled in the first triangle corresponds with the first vertex labeled in the second triangle. The same is true for the other two vertices.

The sides of triangles are line segments and are defined by two vertices. The corresponding sides also depend on the order of the triangle labels.

2. Name the angle pairs and side pairs to find a triangle correspondence that matches sides of equal length and angles of equal measurement.



The tick marks on the triangles' sides indicate correspondence. Corresponding sides have the same number of tick marks.



$AB = DE$ $BC = EF$ $AC = DF$

$\angle A = \angle D$ $\angle B = \angle E$ $\angle C = \angle F$

The arcs in the triangles' angles indicate correspondence. Corresponding angles have the same number of arcs.

$\triangle ABC \leftrightarrow \triangle DEF$

I know the order that I use to name the triangles is important. The letters of the corresponding angles must be in the same position for both triangles.

G7-M6-Lesson 6: Drawing Geometric Shapes

Necessary Tools

Students need a ruler, protractor, and compass to complete the homework assignment.

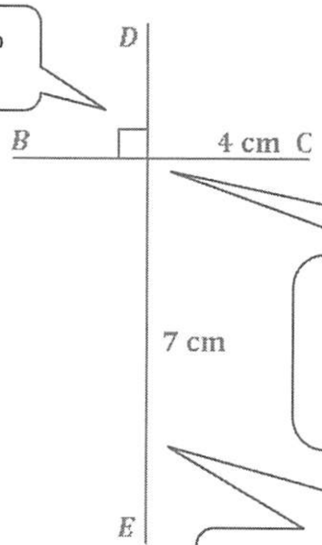
I use rulers to measure and draw line segments, protractors to construct angles, and compasses to draw circles.

Use a ruler, protractor, and compass to complete the following problems.

1. Draw a segment BC that is 4 cm in length, perpendicular to segment DE, which is 7 cm in length.

I know that perpendicular tells me that the two segments will create a right angle.

I first use my ruler to draw segment BC.



Next, I use my protractor to create a $90°$ angle. To use my protractor, I place the origin and base line of my protractor on $\overline{BC}$ and mark where $90°$ is located.

Once I know the size of my angle, I can finally use my ruler to measure 7 cm to draw segment DE.

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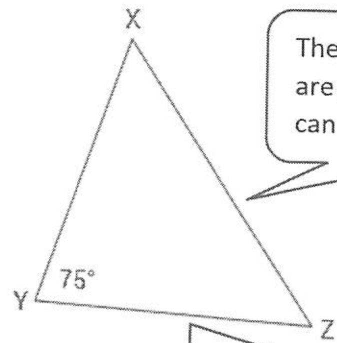
2. Draw $\triangle XYZ$ so that $\angle Y$ has a measurement of $75°$.

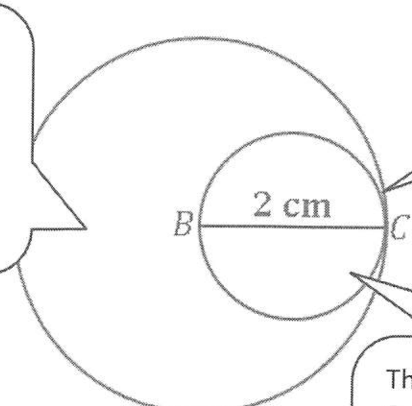


The length of the sides are not specified, so they can be any length.

I draw a line segment so I know where to place the origin and base line of the protractor, and then I use my protractor to create a $75°$ angle for $\angle Y$.

3. Draw a segment BC that is 2 cm in length. Draw a circle with center B and radius BC. Draw a second circle with diameter BC.

In order to draw the larger circle, I set my compass to 2 cm and place the needle at point B. Then, I construct the circle.



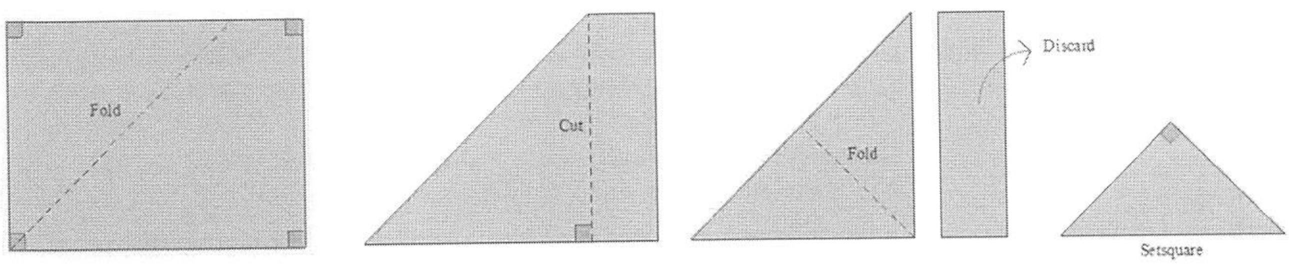
I first use my ruler to construct segment BC.

The smaller circle has a diameter of 2 cm, which makes the radius 1 cm. Therefore, I set my compass to 1 cm, place the pencil at point B, and the needle will create the center of the circle. Then, I construct the circle.

G7-M6-Lesson 7: Drawing Parallelograms

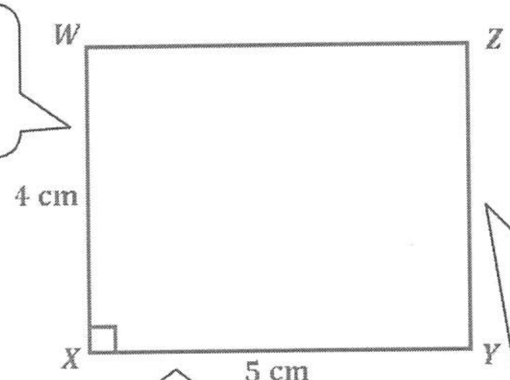
Necessary Tools

Students need a ruler, protractor, and setsquare to complete this homework assignment. Students created setsquares at school, but, if necessary, students can follow the diagrams below to make a new one.



1. Use a setsquare and a ruler to construct rectangle $WXYZ$ with $WX = 4$ cm and $XY = 5$ cm.

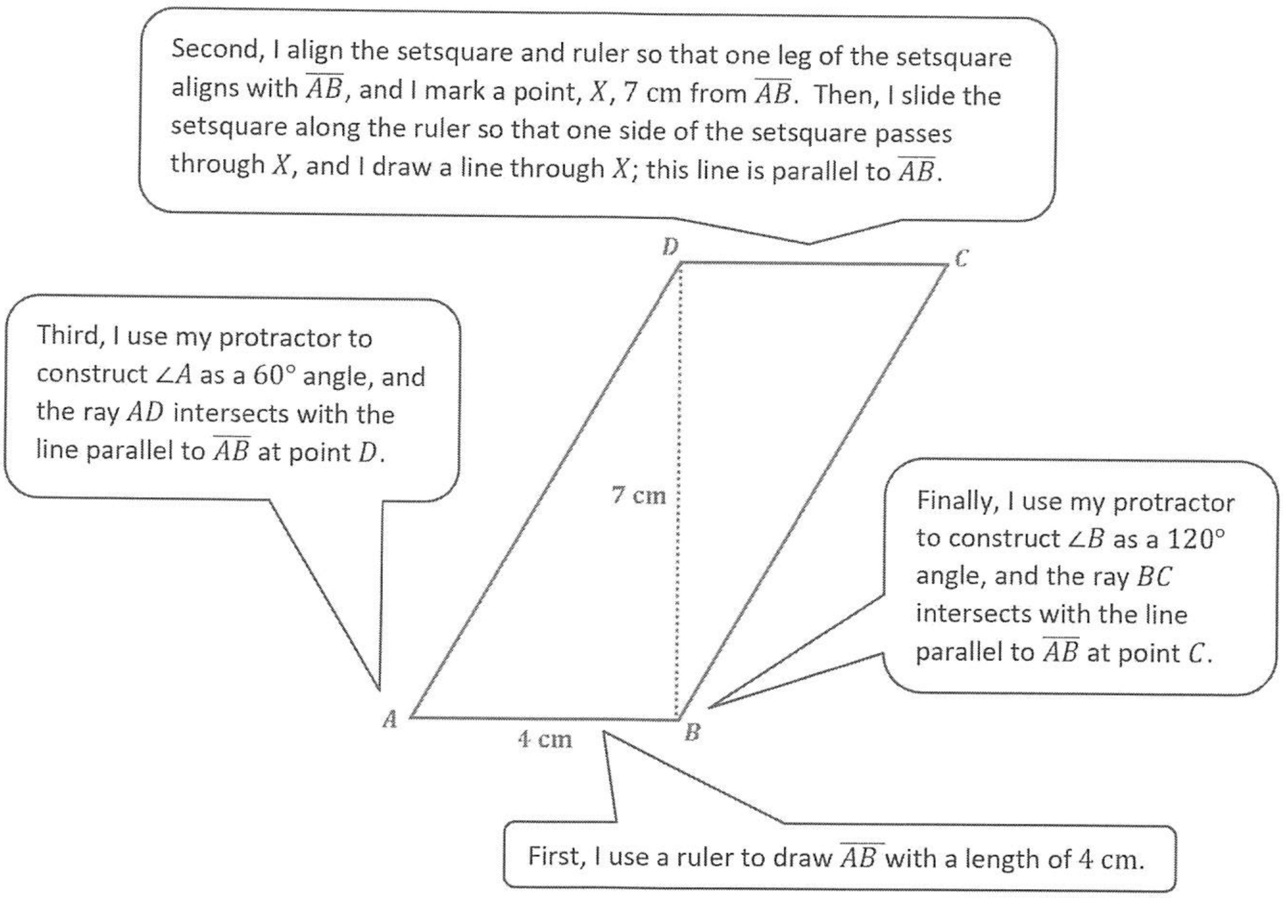
First, I draw segment WX by using my ruler to measure 4 cm.



Second, I use the setsquare to create a segment that is parallel to $\overline{WX}$. To do this, I align one leg of the setsquare with $\overline{WX}$ and place the ruler along the other leg of the setsquare. I make a mark 5 cm away from $\overline{WX}$. I slide the setsquare along the ruler until I reach the mark and use the leg of the setsquare to draw a segment through the mark that is parallel to $\overline{WX}$.

Finally, I need to create right angles and connect the two parallel segments. To do this, I align one leg of my setsquare with $\overline{WX}$, line up my ruler so the outer portion goes through point X, and then I draw a segment. I mark the point that intersects the segment parallel to $\overline{WX}$ as Y. I repeat this process to find point Z.

2. Use a setsquare, ruler, and protractor to draw parallelogram $ABCD$ so that $\angle A = 60°$, $AB = 4$ cm, $\angle B = 120°$, and the altitude to $\overline{AB}$ is 7 cm.



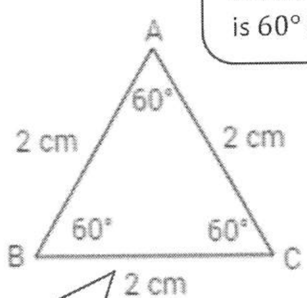
G7-M6-Lesson 8: Drawing Triangles

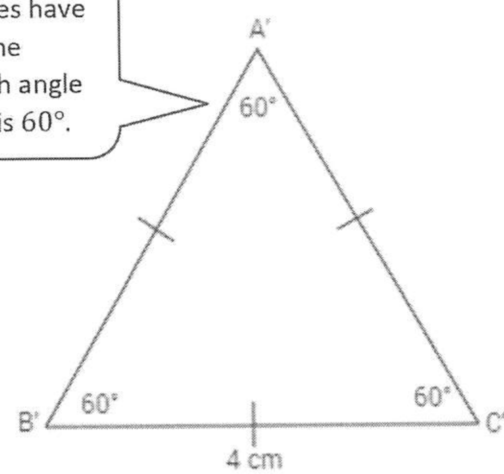
Necessary Tools

Students need a ruler and protractor to complete the homework assignment.

1. Draw two different equilateral triangles, $\triangle ABC$ and $\triangle A'B'C'$. A side length of $\triangle ABC$ is 2 cm. A side length of $\triangle A'B'C'$ is 4 cm. Label all sides and angle measurements. Why are your triangles not identical?

I know equilateral triangles have three angles with the same measure. Therefore, each angle is $60°$ because $180° \div 3$ is $60°$.





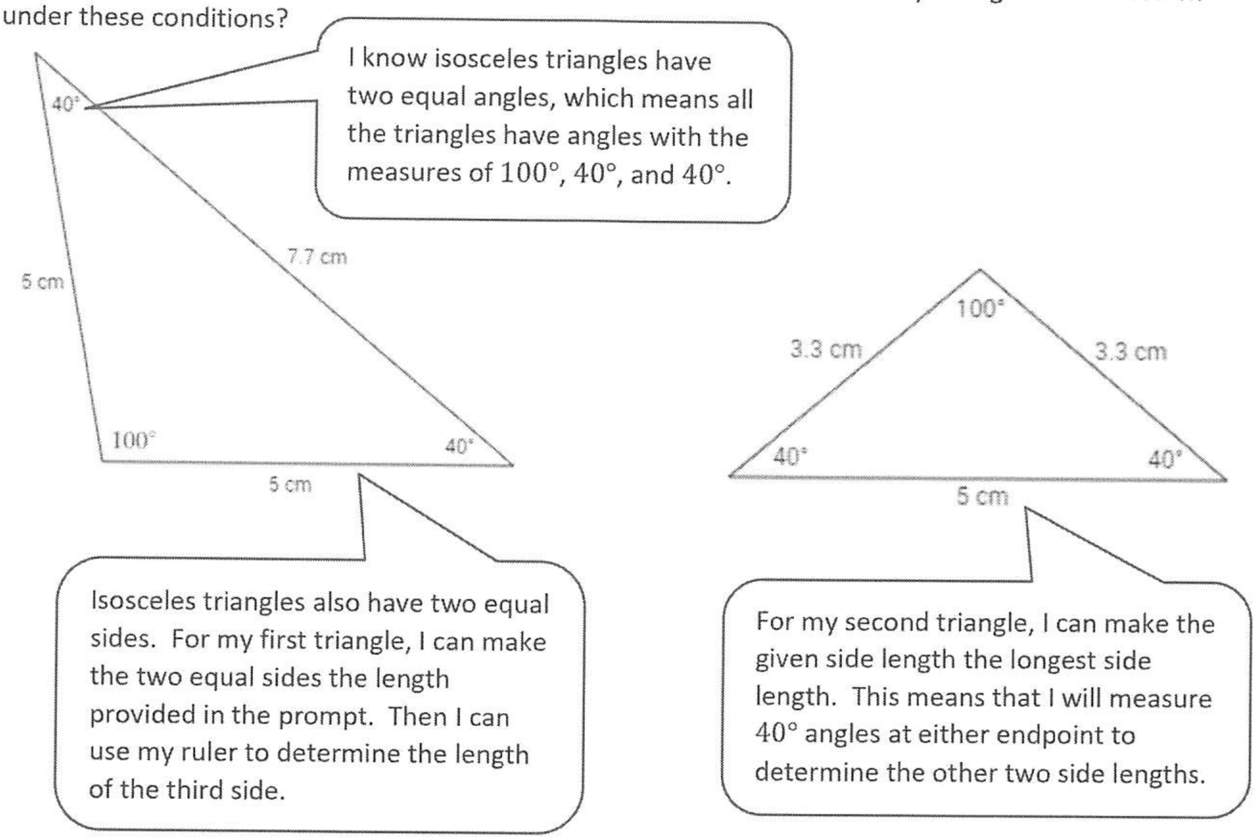
I use my ruler to draw a line segment with a length of 2 cm. Then, I use my protractor to measure $60°$ angles to create my equilateral triangle.

I can use identical tick marks to show that every side has the same length of 4 cm.

Even though the angles are identical in both triangles, the triangles are not identical because there is no correspondence that matches equal sides to equal sides.

I know that identical triangles must have three identical angles *and* three identical sides.

2. Draw all the isosceles triangles that satisfy the following conditions: one angle measure is $100°$ and one side has a length of 5 cm. Label all angle and side measurements. How many triangles can be drawn under these conditions?

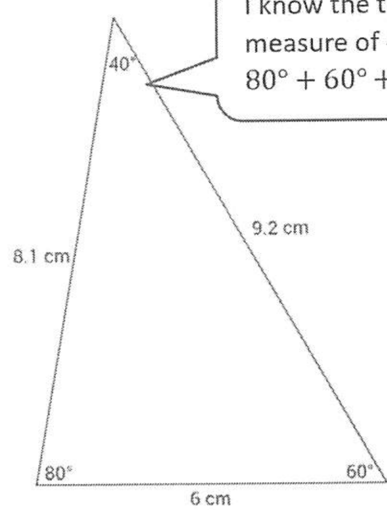


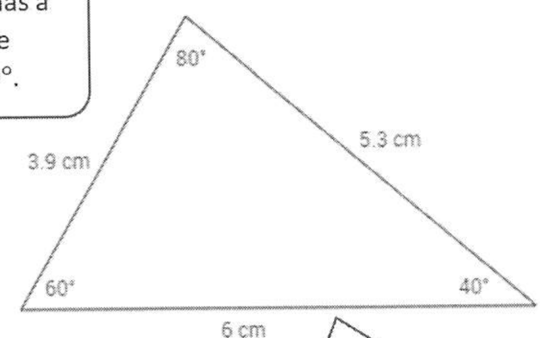
Only two triangles can be created under these conditions.

3. Draw three non-identical triangles so that two angles measure 60° and 80° and one side measures 6 cm. Why are these triangles not identical?

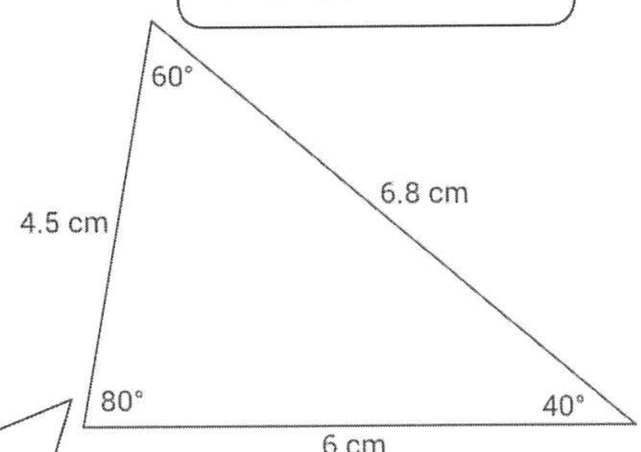
Even though there is a correspondence that will match equal angles to equal angles, these triangles are not identical because there is no correspondence that will match equal sides to equal sides.

I know the third angle has a measure of 40° because $80^\circ + 60^\circ + 40^\circ$ is 180°.





For each triangle, I can start by drawing a side with a measurement of 6 cm.



I can measure an angle on each vertex of the first side length I draw. Then, I draw a line segment from each endpoint until the segments intersect at the third vertex. I can use my ruler to determine the length of these two side lengths.

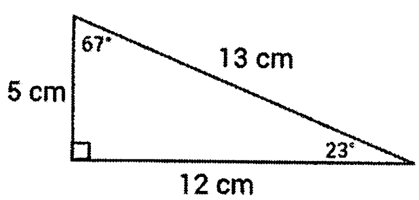
G7-M6-Lesson 9: Conditions for a Unique Triangle—Three Sides and Two Sides and the Included Angle

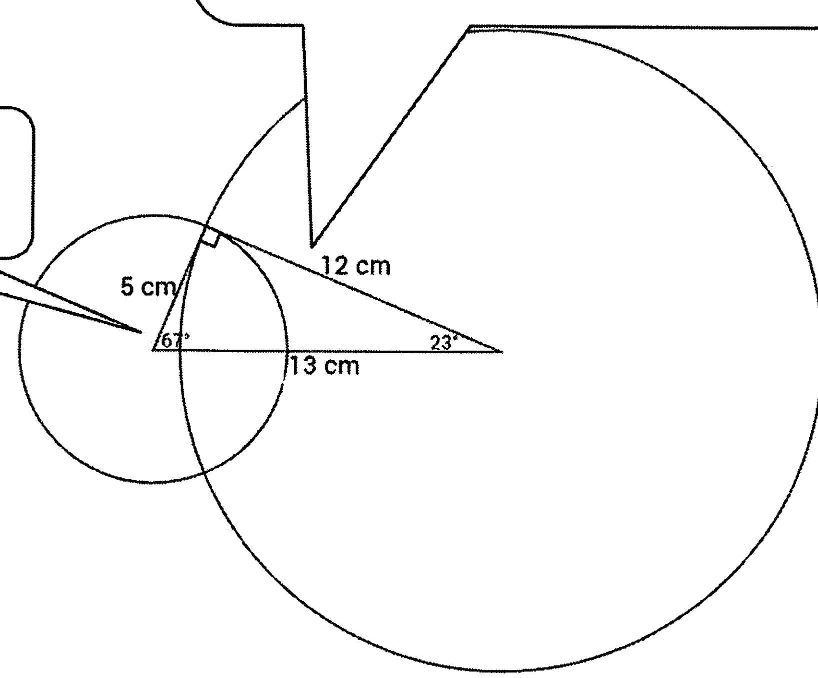
Necessary Tools

Students need a ruler, protractor, and compass to complete this homework assignment.

1. A triangle with side lengths 5 cm, 12 cm, and 13 cm is shown below. Use your compass and ruler to draw a triangle with the same side lengths. Leave all construction marks as evidence of your work, and label all side and angle measurements.

   Under what condition is the triangle drawn? Compare the triangle you drew to the triangle shown below. Are the triangles identical? Did the condition determine a unique triangle? Use your construction to explain why.



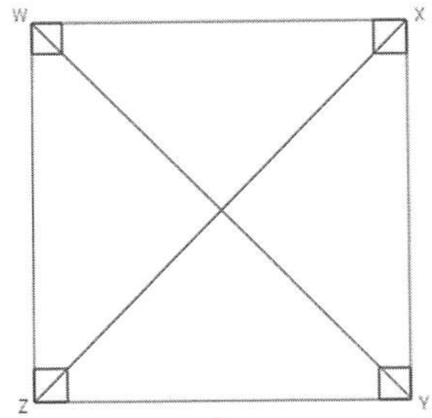


The triangle I constructed is identical to the given triangle. Even though there are two options for the third vertex, each option results in the same triangle. Therefore, the three sides condition determines a unique triangle.

2. Diagonals $\overline{WY}$ and $\overline{XZ}$ are drawn in square $WXYZ$. Show that $\triangle XYZ$ is identical to $\triangle YZW$, and then use this information to show that the diagonals are equal in length.

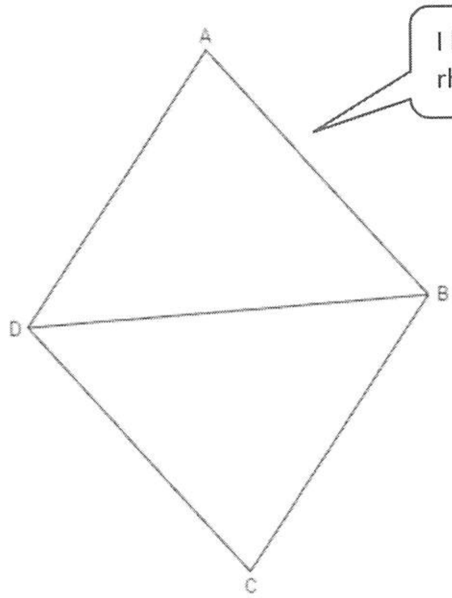
I can use the two sides and an included angle condition to show that $\triangle XYZ$ is identical to $\triangle YZW$. I know corresponding sides $\overline{XY}$ and $\overline{YZ}$ are the same length because they are both sides of the same square. For the same reason, corresponding sides $\overline{YZ}$ and $\overline{ZW}$ are also the same length. $\angle XYZ$ and $\angle YZW$ are both right angles, so they have the same measure.

Since these two triangles are identical, I know each pair of corresponding sides must be the same length. Therefore, the diagonals (the third side of each triangle) must be the same length.



I know that the four sides of a square are all the same length. I also know that all four angles are right angles.

3. Diagonal $\overline{BD}$ is drawn in rhombus $ABCD$. Describe the condition that can be used to justify that $\triangle ABD$ is identical to $\triangle CBD$. Can you conclude that the measures of $\angle ABD$ and $\angle CBD$ are the same? Support your answer with a diagram and an explanation of the correspondence(s) that exists.



I know all four sides of a rhombus have the same length.

I can use the three sides condition to show that $\triangle ABD$ is identical to $\triangle CBD$. I know that corresponding sides $\overline{AB}$ and $\overline{CB}$ are the same length because they are both sides of the same rhombus. For the same reason, the corresponding sides $\overline{AD}$ and $\overline{CD}$ are also the same length. The third sides of each triangle ($\overline{BD}$) are the same length because they are the same line segment.

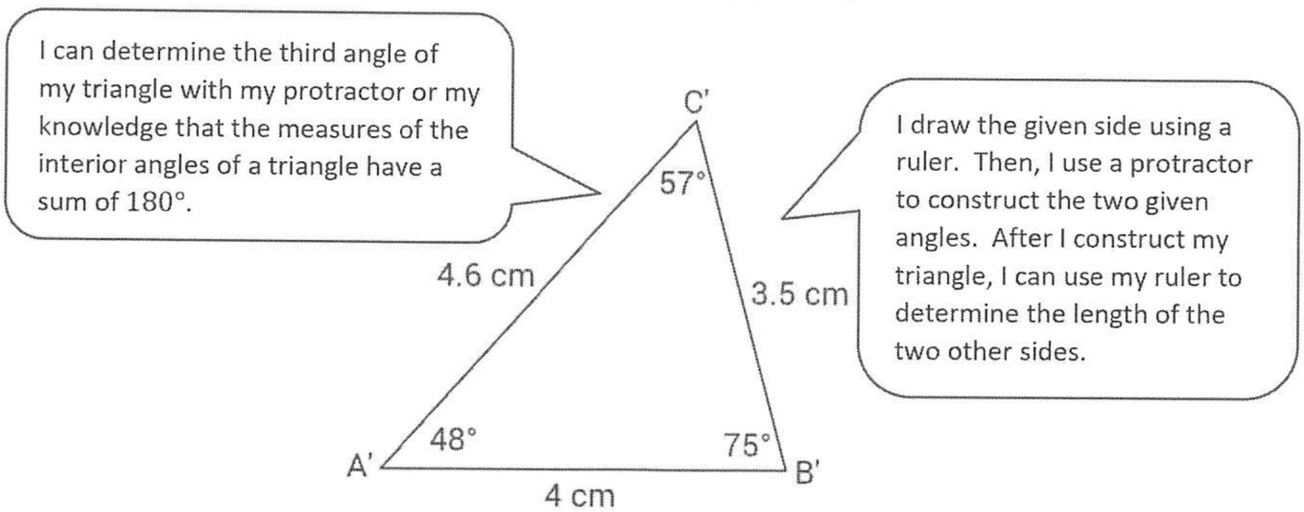
$\angle ABD$ and $\angle CBD$ have the same measure because they are corresponding angles of two identical triangles.

If two triangles are identical, I know there are corresponding angles that have the same measure.

G7-M6-Lesson 10: Conditions for a Unique Triangle—Two Angles and a Given Side

1. In $\triangle ABC$, $\angle A = 48°$, $\angle B = 75°$, and $AB = 4$ cm. Draw $\triangle A'B'C'$ under the same condition as $\triangle ABC$. Label all side and angle measurements.

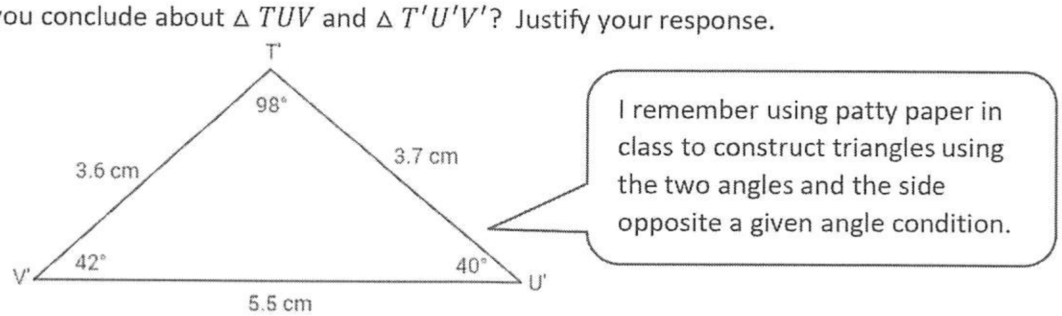
   What can you conclude about $\triangle ABC$ and $\triangle A'B'C'$? Justify your response.



Since both triangles are drawn under the same condition, and the two angles and included side condition determines a unique triangle, then both triangles determine the same unique triangle. Therefore, $\triangle ABC$ and $\triangle A'B'C'$ are identical.

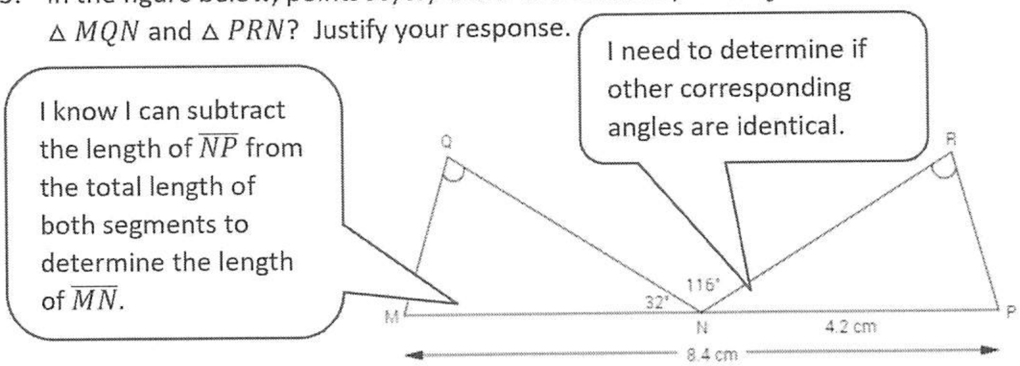
2. In $\triangle TUV$, $\angle V = 42°$, $\angle T = 98°$, and $UV = 5.5$ cm. Draw $\triangle T'U'V'$ under the same condition as $\triangle TUV$. Label all side and angle measurements.

   What can you conclude about $\triangle TUV$ and $\triangle T'U'V'$? Justify your response.



Since both triangles are drawn under the same condition, and the two angles and the side opposite a given angle condition determines a unique triangle, then both triangles determine the same unique triangle. Therefore, $\triangle TUV$ and $\triangle T'U'V'$ are identical.

3. In the figure below, points M, N, and P are collinear, and $\angle Q = \angle R$. What can be concluded about $\triangle MQN$ and $\triangle PRN$? Justify your response.



Let x represent the measure of $\angle RNP$.

I know from previous lessons that the measures of angles on a line have a sum of $180°$.

$$32° + 116° + x = 180°$$
$$148° + x = 180°$$
$$148° - 148° + x = 180° - 148°$$
$$x = 32°$$

The measure of $\angle RNP$ is $32°$. This means corresponding angles $\angle RNP$ and $\angle QNM$ have the same measure.

Let y represent the measure of $\overline{MN}$.

$$y + 4.2\text{ cm} = 8.4\text{ cm}$$
$$y + 4.2\text{ cm} - 4.2\text{ cm} = 8.4\text{ cm} - 4.2\text{ cm}$$
$$y = 4.2\text{ cm}$$

The measure of $\overline{MN}$ is 4.2 cm. This means the corresponding sides $\overline{MN}$ and $\overline{PN}$ are the same length.

The diagram indicates $\angle Q$ has the same measure as $\angle R$.

Therefore, $\triangle MQN$ and $\triangle PRN$ are identical because the same measurements in both triangles satisfy the two angles and the side opposite a given angle condition, which means they both determine the same unique triangle.

G7-M6-Lesson 11: Conditions on Measurements That Determine a Triangle

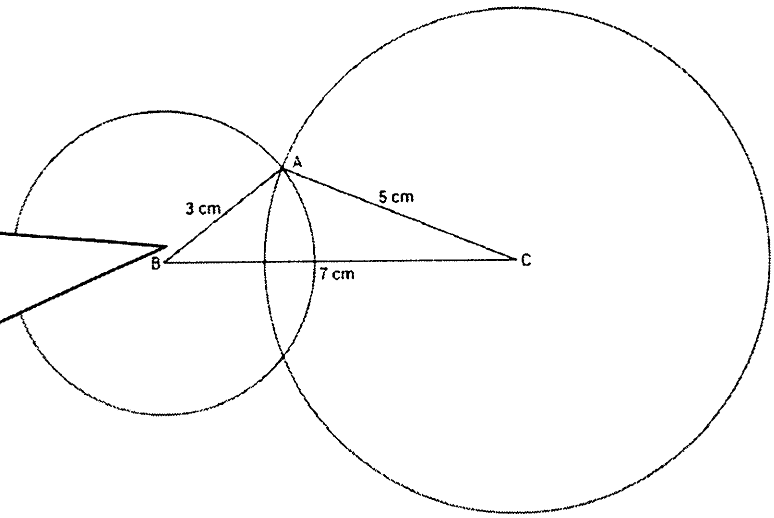
Necessary Tools

Students need a ruler and compass to complete the homework assignment.

1. Decide whether each set of three given lengths determines a triangle. For any set of lengths that does determine a triangle, use a ruler and compass to draw the triangle. Label all side lengths. For sets of lengths that do not determine a triangle, write "Does not determine a triangle," and justify your response.

   a. 3 cm, 5 cm, 7 cm

I use my ruler to draw the longest side. On one endpoint, I use my compass to draw a circle with a radius of 3 cm. I construct a circle with a radius of 5 cm on the other endpoint. One of the intersection points of these two circles is the third vertex of my triangle.



   b. 6 cm, 13 cm, 5 cm

   ***These side lengths do not determine a triangle because the two shortest side lengths are too short to create a triangle with a side length of* 13 cm.**

I know that three lengths determine a triangle if the largest length is less than the sum of the other two lengths.

$$6 \text{ cm} + 5 \text{ cm} \ngtr 13 \text{ cm}$$

Therefore, these sides lengths will not form a triangle.

2. For each angle measurement below, provide one angle measurement that will determine a triangle and one that will not determine a triangle. Assume that the angles are being drawn to a horizontal segment XY; describe the position of the non-horizontal rays of $\angle X$ and $\angle Y$.

$\angle X$	$\angle Y$: A Measurement That Determines a Triangle	$\angle Y$: A Measurement That Does Not Determine a Triangle
$50°$	$40°$	$130°$
$120°$	$59°$	$61°$

In order for the angles to determine a triangle, the sum of the measures of the two given angles must be less than $180°$.

If the sum of the measures of two angles is greater than or equal to $180°$, the angles do not determine a triangle because the non-horizontal rays do not intersect.

3. For the given lengths, provide the minimum and maximum whole number side lengths that determine a triangle.

Given Side Lengths	Minimum Whole Number Third Side Length	Maximum Whole Number Third Side Length
4 cm, 5 cm	2 cm	8 cm
2 cm, 14 cm	13 cm	15 cm

I can calculate the minimum possible third length by making the sum of the lengths of the third side and the shorter given side one whole number more than the length of the longest side.

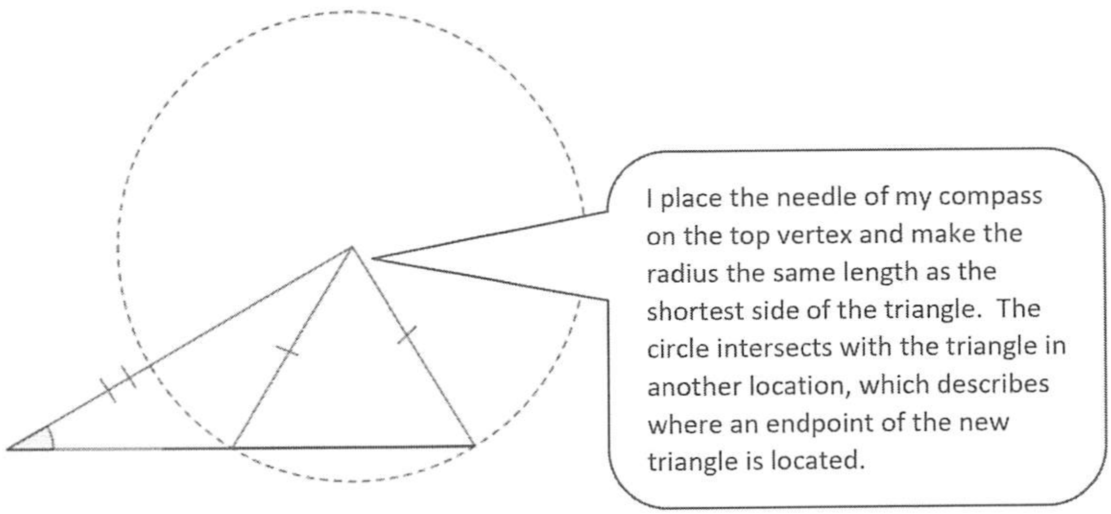
I can calculate the maximum possible third length by calculating the sum of the two given side lengths. I know the length of the third side must be one whole number less than this sum.

G7-M6-Lesson 12: Unique Triangles—Two Sides and a Non-Included Angle

Necessary Tools

Students need a compass to complete the homework assignment.

1. In the triangle below, two sides and a non-included angle are marked. Use a compass to draw a non-identical triangle that has the same measurements as the marked angle and marked sides. Draw the new triangle on top of the old triangle. What is true about the marked angle in each triangle that results in two non-identical triangles under this condition?



I was able to create a non-identical triangle with the same measurements as the marked angle and marked sides because the non-included angle is an acute angle.

I know that if the non-included angle is smaller than 90°, the two sides and non-included angle condition does not determine a unique triangle.

2. A sub-condition of the two sides and non-included angle condition is provided in each row of the following table. Decide whether the information determines a unique triangle. Answer with a *yes, no*, or *maybe* (for a case that may or may not determine a unique triangle).

I know that the two sides and non-included angle condition always determines a unique triangle when the non-included angle is $90°$ or larger.

	Condition	Determines a Unique Triangle?
1	Two sides and a non-included $100°$ angle	*Yes*
2	Two sides and a non-included $85°$ angle	*Maybe*
3	Two sides and a non-included $45°$ angle, where the side adjacent to the angle is longer than the side opposite the angle	*No*

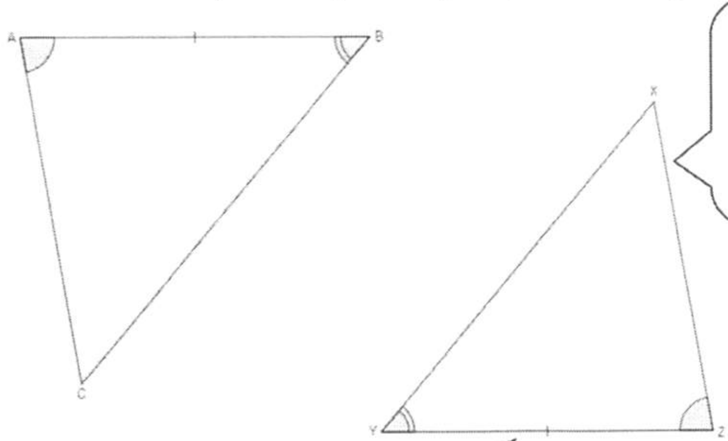
I know that the two sides and non-included angle condition sometimes determines a unique triangle when the non-included angle is acute.

In order for the two sides and non-included angle condition to determine a unique triangle when the non-included angle is acute, the side adjacent to the angle must be shorter than the side opposite the angle.

G7-M6-Lesson 13: Checking for Identical Triangles

In each of the following problems, two triangles are given. State whether the triangles are *identical, not identical,* or *not necessarily identical.* If the triangles are identical, give the triangle conditions that explain why, and write a triangle correspondence that matches the sides and angles. If the triangles are not identical, explain why. If it is not possible to definitively determine whether the triangles are identical, write "the triangles are not necessarily identical," and explain your reasoning.

1.

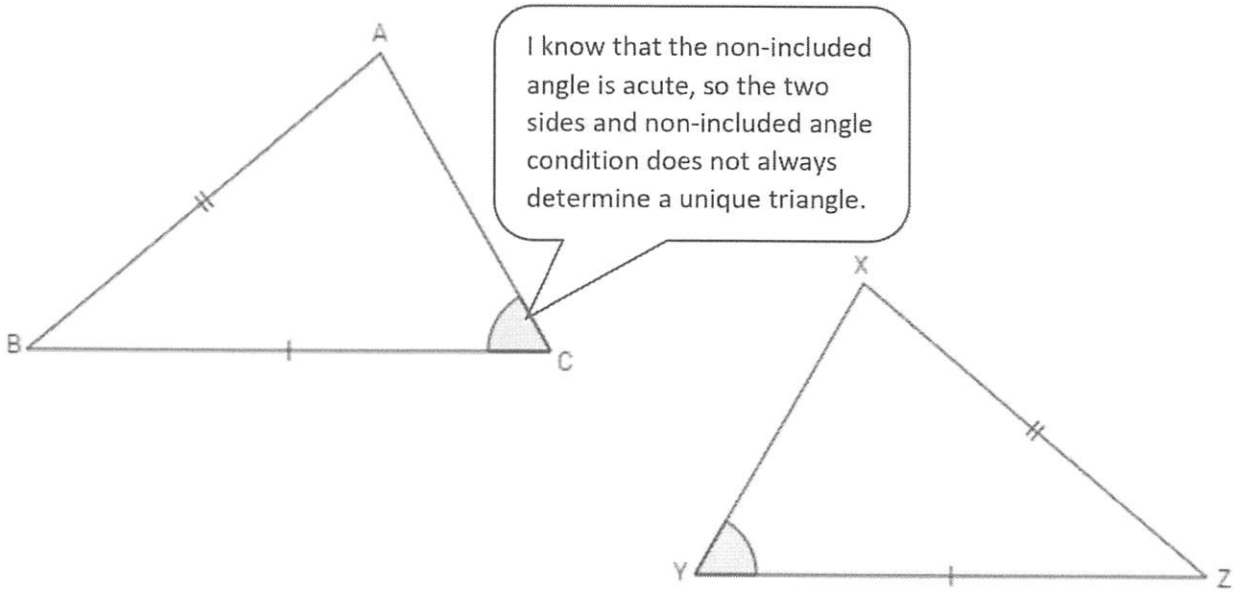


I see two sets of identical angles and one set of identical sides. The location of the angles and sides follow the two angles and included side condition.

I know that $\angle A$ corresponds to $\angle Z$, and $\angle B$ corresponds to $\angle Y$. I also know that side $\overline{AB}$ corresponds to $\overline{ZY}$. I can use this information to write the correspondence.

The triangles are identical by the two angles and included side condition. The correspondence $\triangle ABC \leftrightarrow \triangle ZYX$ matches two equal pairs of angles and one equal pair of included sides. Since both triangles have parts under the condition of the same measurement, the triangles must be identical.

2.

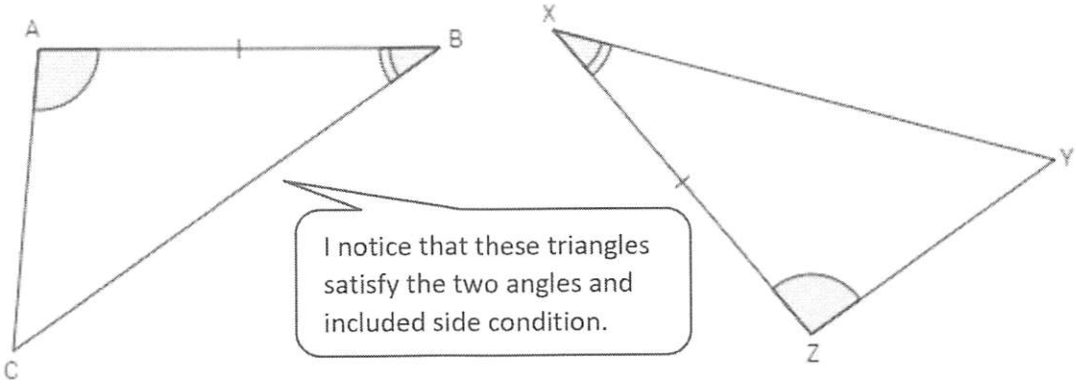


The triangles are not necessarily identical by the two sides and a non-included angle condition. I would need more information about the given sides in order to determine whether or not the two triangles are identical.

I know that the side adjacent to the given angle must be shorter than the side opposite the given angle in order for these triangles to be identical. However, I cannot determine which side is longer with the information given.

In the following problems, three pieces of information are given for $\triangle ABC$ and $\triangle XYZ$. Draw, freehand, the two triangles (do not worry about scale), and mark the given information. If the triangles are identical, give a triangle correspondence that matches equal angles and equal sides. Explain your reasoning.

3. $\angle A = \angle Z$, $\angle B = \angle X$, and $AB = ZX$



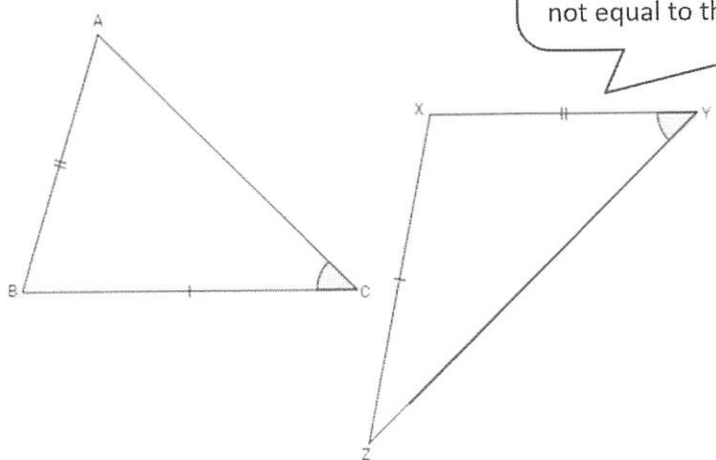
> I notice that these triangles satisfy the two angles and included side condition.

These triangles are identical by the two angles and included side condition. The triangle correspondence $\triangle ABC \leftrightarrow \triangle ZXY$ matches the two pairs of equal angles and one pair of equal sides. Since both triangles have parts under the condition of the same measurement, the triangles must be identical.

> When I write the correspondence, I need to make sure the equal angles and equal sides are in the same location for each triangle.

4. $AB = XY$, $BC = XZ$, and $\angle C = \angle Y$

> Originally I thought these triangles satisfied the two sides and non-included angle condition. However, I notice that the side adjacent to $\angle C$ is not equal to the side that is adjacent to $\angle Y$.

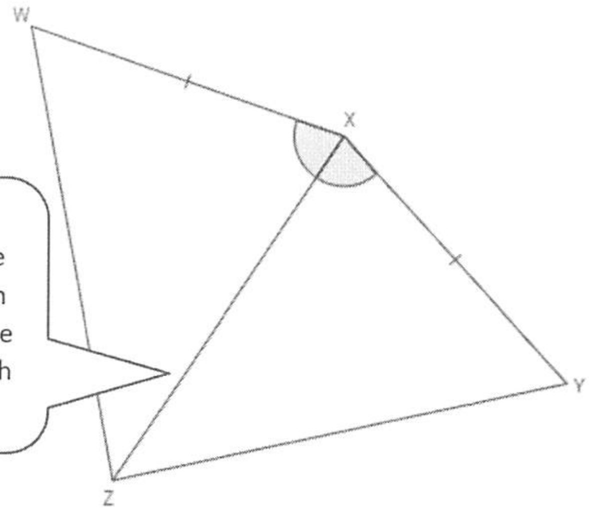


These triangles are not necessarily identical. In $\triangle ABC$, the marked angle is adjacent to the side marked with one tick mark. In $\triangle XYZ$, the marked angle is adjacent to the side marked with two tick marks. Since the sides adjacent to the equal angles are not equal in length, the triangles do not fit any of the conditions that determine a unique triangle.

G7-M6-Lesson 14: Checking for Identical Triangles

In the following problems, determine whether the triangles are *identical, not identical,* or *not necessarily identical*; justify your reasoning. If the relationship between the two triangles yields information that establishes a condition, describe the information. If the triangles are identical, write a triangle correspondence that matches the sides and angles.

1.

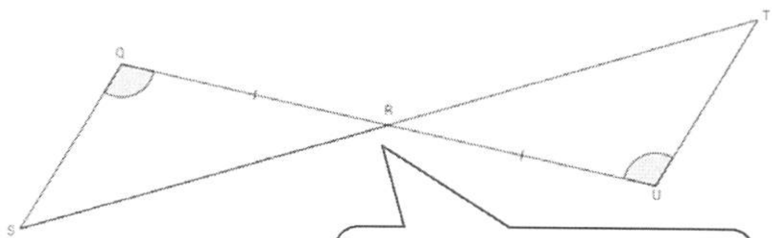


The two triangles share side $\overline{XZ}$, which means this side is equal in both triangles.

These triangles are identical by the two sides and the included angle condition. The triangle correspondence $\triangle WXZ \leftrightarrow \triangle YXZ$ matches two pairs of equal sides and one pair of equal angles.

One of the equal pairs of sides is shared side $\overline{XZ}$.

2.

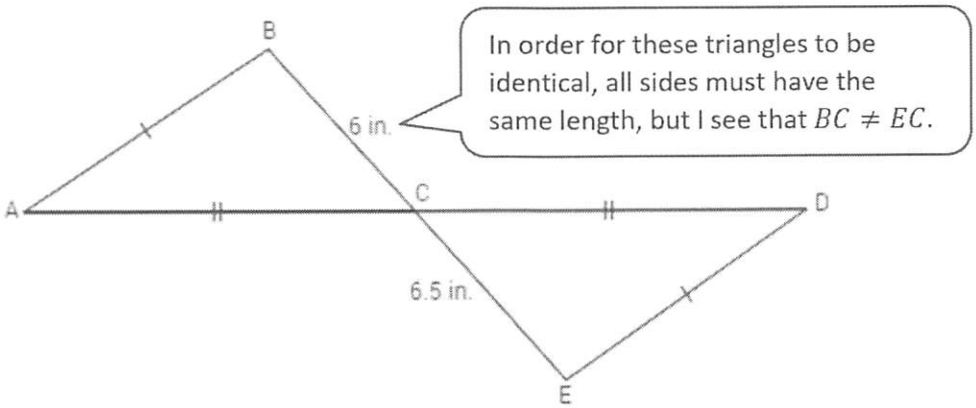


I know $\angle QRS$ and $\angle URT$ have the same measure because they are vertical angles.

The two triangles are identical by the two angles and the included side condition. The triangle correspondence $\triangle QRS \leftrightarrow \triangle URT$ matches the two pairs of equal angles and one pair of equal sides.

One of the pairs of equal angles are vertical angles.

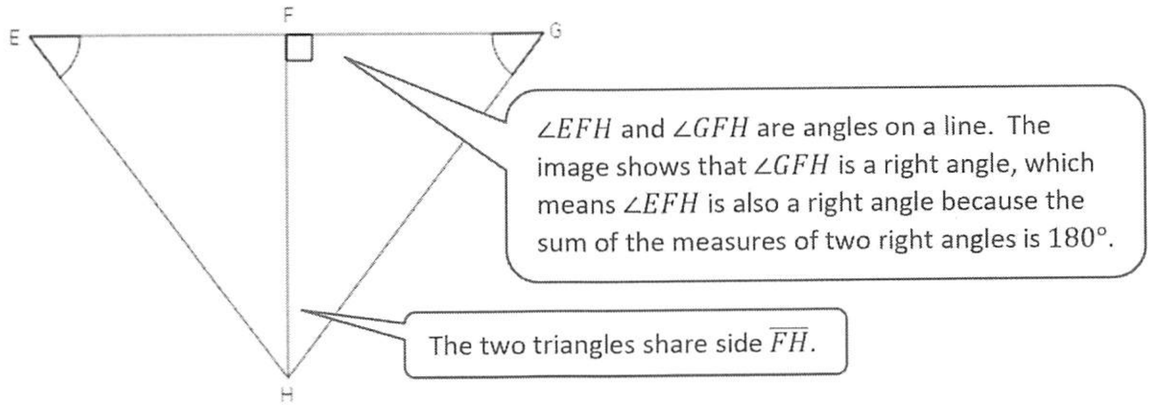
3.



The two triangles are not identical because the correspondence that matches the two marked equal pairs of sides also matches sides $\overline{BC}$ and $\overline{EC}$, which are not equal in length.

The triangles are not identical, so I cannot write a triangle correspondence.

4.



These two triangles are identical by the two angles and side opposite a given angle condition. The correspondence $\triangle EFH \leftrightarrow \triangle GFH$ matches the two pairs of equal angles and the one pair of equal sides.

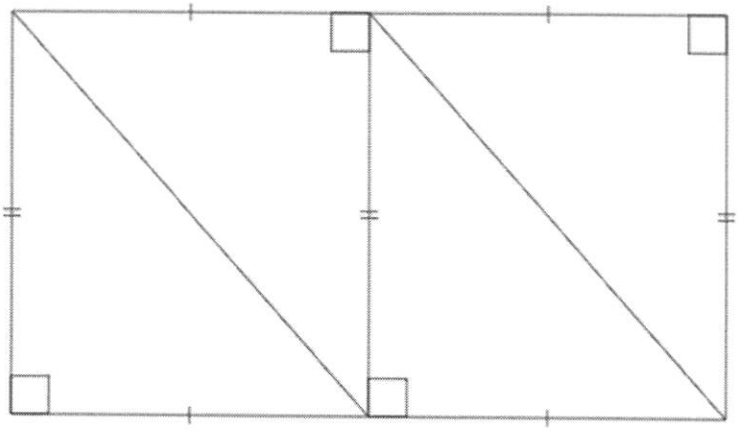
One of the pairs of equal angles is $\angle EFH$ and $\angle GFH$ because they are both right angles.

G7-M6-Lesson 15: Using Unique Triangles to Solve Real-World and Mathematical Problems

1. Ms. Thompson wants to cut different sheets of paper into four equal triangles for a class activity. She first cuts the paper into equal halves in the shape of rectangles, and then she cuts each rectangle along a diagonal.

   Did Ms. Thompson cut the paper into 4 equal pieces? Explain.

Each of the four triangles have two sides and one right angle from the rectangles from the first cut. Due to properties of rectangles, I know there is a correspondence between all four triangles that matches two pairs of equal sides and one pair of equal angles.

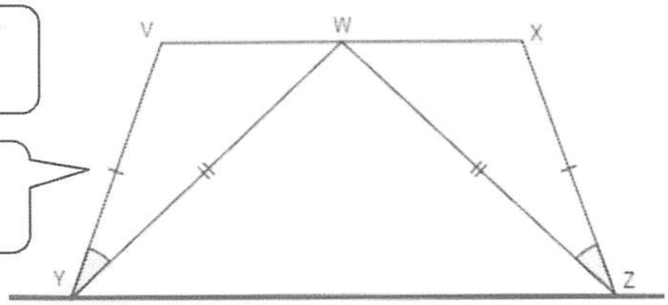


Ms. Thompson did cut the piece of paper into four identical triangles. The first cut Ms. Thompson made resulted in two equal rectangles, which means the corresponding sides have the same length. I also know that all four angles in a rectangle are right angles. Therefore, the four triangles are identical due to the two sides and an included angle condition.

2. The bridge below, which crosses a road, is built out of two triangular supports. The point W lies on $\overline{VX}$. The beams represented by $\overline{YW}$ and $\overline{ZW}$ are equal in length, and the beams represented by $\overline{YV}$ and $\overline{ZX}$ are equal in length. If the supports were constructed so that $\angle Y$ and $\angle Z$ are equal in measurement, is point W the midpoint of $\overline{VX}$? Explain.

If W is the midpoint of $\overline{VX}$, then $\overline{VW}$ and $\overline{XW}$ must have the same length.

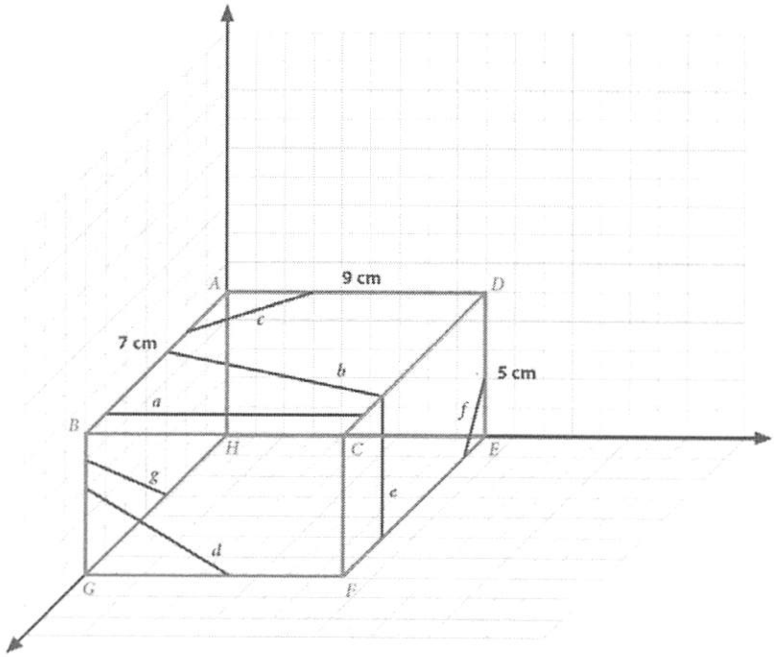
I add marks on the image from the information provided in the prompt.



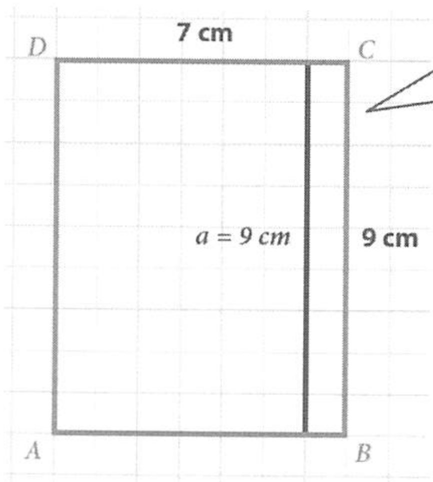
Yes, W is the midpoint of $\overline{VX}$. I know $\triangle WVY$ and $\triangle WXZ$ are identical triangles due to the two sides and included angle condition. If these two triangles are identical, that means the corresponding sides have the same length. Therefore, $\overline{VW}$ and $\overline{XW}$ are the same length.

G7-M6-Lesson 16: Slicing a Right Rectangular Prism with a Plane

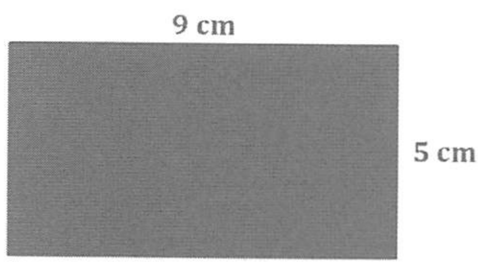
A right rectangular prism is shown along with line segments that lie in a face. For segments a and b, draw and give the approximate dimensions of the slice that results when the slicing plane contains the given line segment and is perpendicular to the face that contains the line segment.



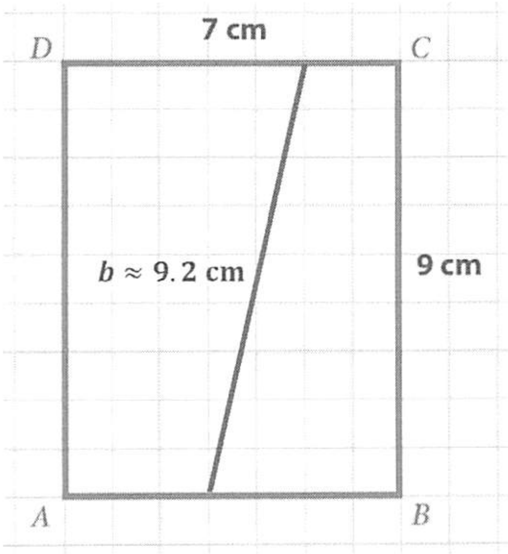
a.



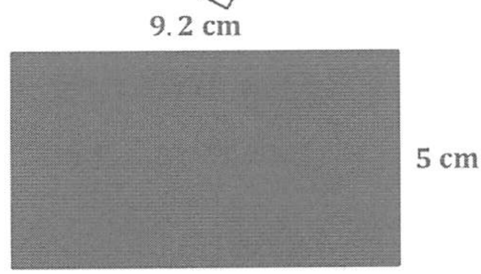
I see that the line segment a is perpendicular to $DCFE$ and $ABGH$. This means that a rectangular slice will be created. I know the dimensions will match the dimensions of different lengths of the prism, in this case 9 cm by 5 cm.



b.



Although the result of slice b is still a rectangle, I need to calculate one of the side lengths.

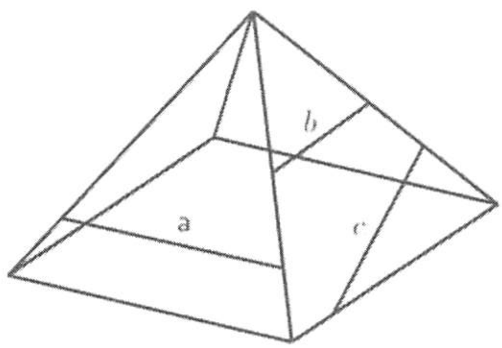


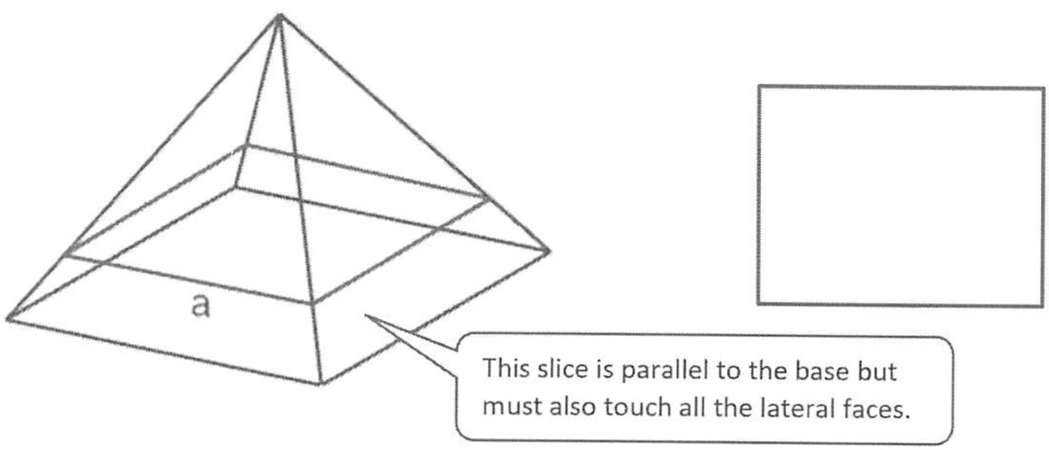
G7-M6-Lesson 17: Slicing a Right Rectangular Pyramid with a Plane

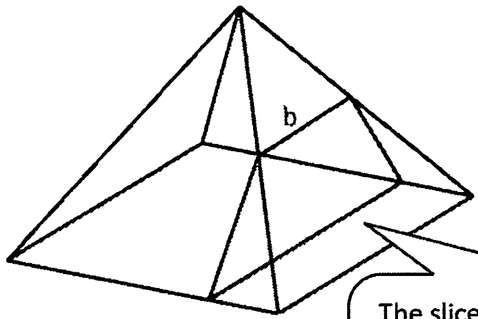
A side view of a right rectangular pyramid is given. The line segments lie in the lateral faces.

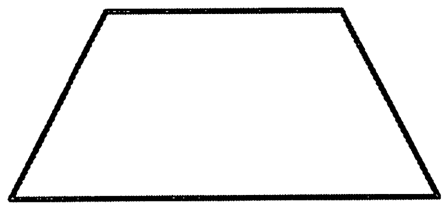
a. For segment a, sketch the resulting slice from slicing the right rectangular pyramid with a slicing plane that contains the segment and is parallel to the base.

b. For segments b and c, sketch the resulting slice from slicing the right rectangular pyramid with a slicing plane that contains the line segment and is perpendicular to the base.

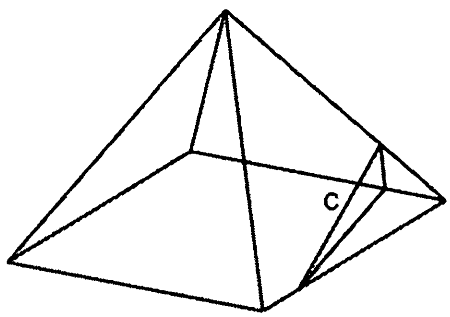


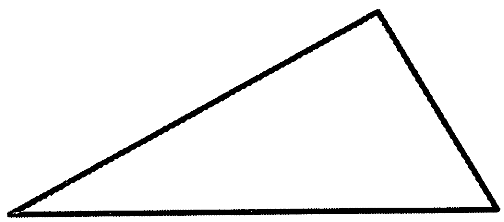






The slice must touch the lateral faces and be perpendicular to the base.
I can also attempt to draw the slice within the prism for a challenge.

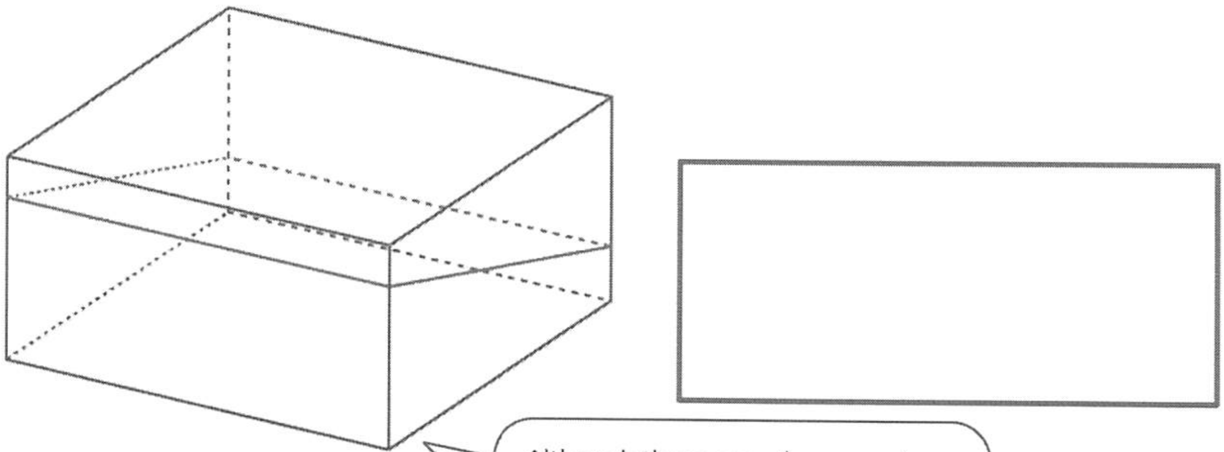




G7-M6-Lesson 18: Slicing on an Angle

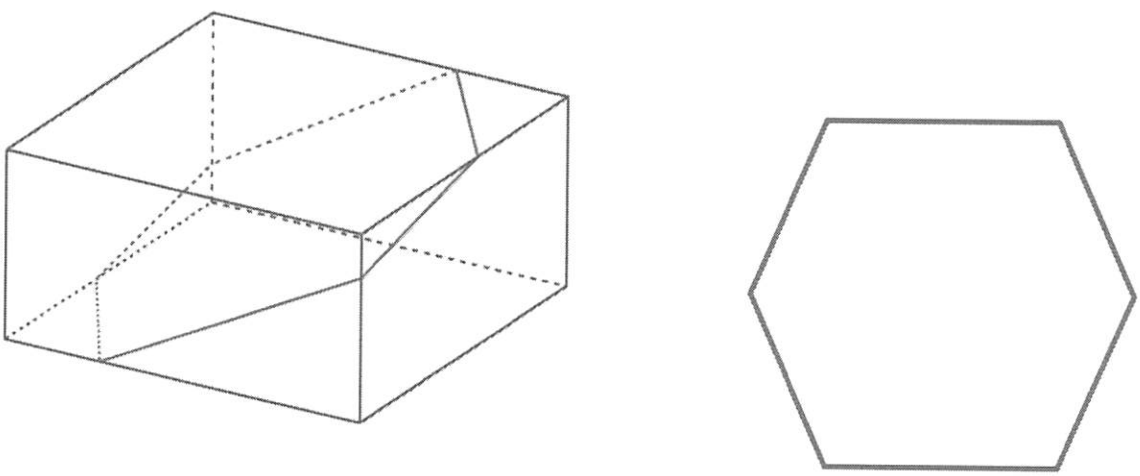
1. Draw a slice into a right rectangular prism at an angle in the form of the provided shape, and draw each slice as a 2D shape.

   a. A quadrilateral

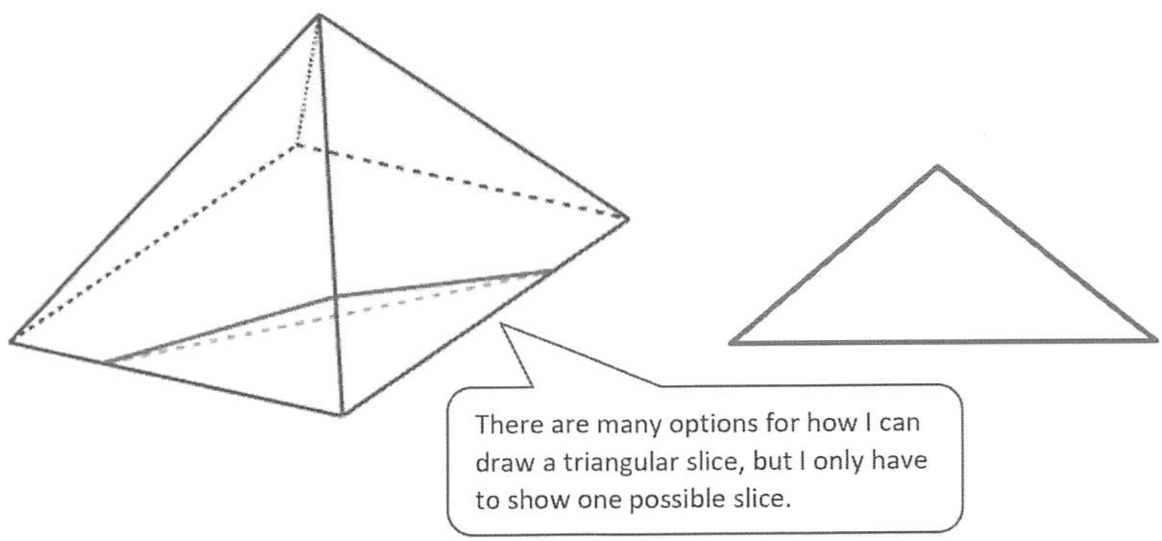


Although there are other ways to draw each slice, I know the vertices of the slice must lie on the prism, and each side of the slice must run along a face of the prism.

   b. A hexagon



2. Draw a slice on an angle into the right rectangular pyramid below in the form of a triangle, then draw the slice as a 2D shape.



3. What other types of shapes can be drawn as a slice in a pyramid?

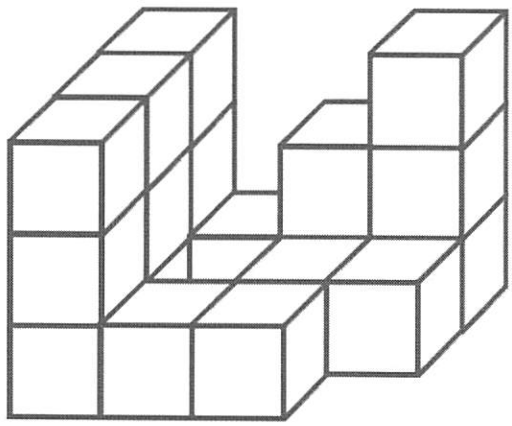
I can draw quadrilateral and pentagonal slices in a pyramid. I cannot draw a slice with more than five sides because there are only five faces on a pyramid.

G7-M6-Lesson 19: Understanding Three-Dimensional Figures

In the given three-dimensional figures, unit cubes are stacked exactly on top of each other on a tabletop. Each block is either visible or below a visible block.

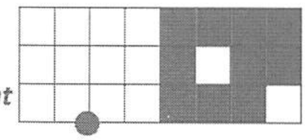
1.

a. The following three-dimensional figure is built on a tabletop. If slices parallel to the tabletop are taken of this figure, then what would each slice look like?



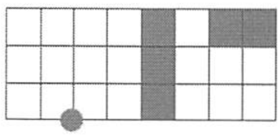
Level 1

Reference Point



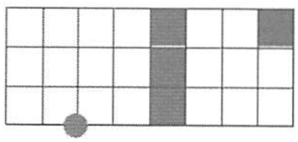
Level 2

Reference Point



Level 3

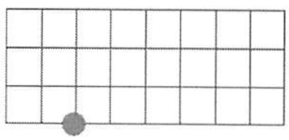
Reference Point



Each reference point shows the cubes that exist in each layer of the figure.

Level 4 and higher

Reference Point



b. Given the level slices in the figure, how many cubes are in the figure?

Level 1: There are 10 *cubes between Level 0 and Level 1.*

Level 2: There are 5 *cubes between Level 1 and Level 2.*

Level 3: There are 4 *cubes between Level 2 and Level 3.*

The total number of cubes in the solid is 19.

I know the number of unit cubes can be determined by counting the shaded squares in Levels 1 to 3.

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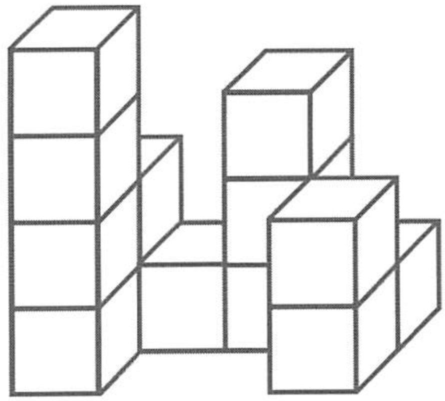
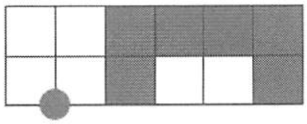
2.

a. The following three-dimensional figure is built on a tabletop. If slices parallel to the tabletop are taken of this figure, then what would each slice look like?



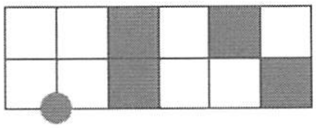
Level 1

Reference Point



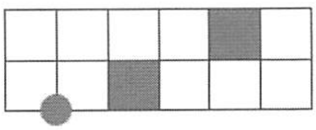
Level 2

Reference Point



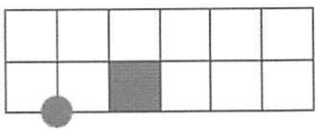
Level 3

Reference Point



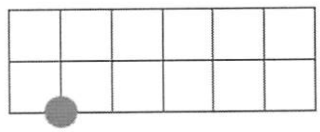
Level 4

Reference Point



Level 5 and higher

Reference Point



b. Given the level slices in the figure, how many cubes are in the figure?

Level 1: There are 6 cubes between Level 0 and Level 1.

Level 2: There are 4 cubes between Level 1 and Level 2.

Level 3: There are 2 cubes between Level 2 and Level 3.

Level 4: There is 1 cube between Level 3 and Level 4.

The total number of cubes in the solid is 13.

This time I have four levels of cubes to add together.

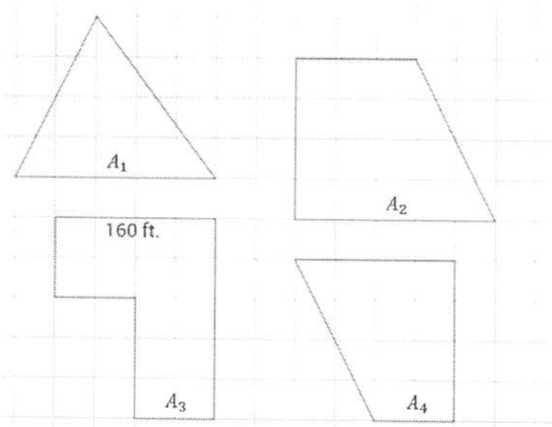
3. When drawing different reference points, why do we not include Level 0?

Level 0 and Level 1 represent the same cubes: Level 0 represents the bottom of these cubes, and Level 1 represents the top of the same cubes. If we showed both Level 0 and Level 1, we would count the same cubes twice.

When I create the reference point drawings, I am showing the top of each cube.

G7-M6-Lesson 20: Real-World Area Problems

1. A farmer has four pieces of unfenced land as shown below in the scale drawing where the dimensions of one side are given. The farmer trades all of the land and $5,000 for 3 acres of similar land that is fenced. If one acre is equal to $43{,}560\text{ ft}^2$, how much per square foot for the extra land did the farmer pay rounded to the nearest cent?



$A_1 = \frac{1}{2}(5\text{ units} \times 4\text{ units})$

$A_1 = 10\text{ square units}$

$A_2 = (3\text{ units} \times 4\text{ units}) + \frac{1}{2}(2\text{ units} \times 4\text{ units})$

$A_2 = 12\text{ square units} + 4\text{ square units}$

$A_2 = 16\text{ square units}$

$A_3 = (4\text{ units} \times 5\text{ units}) - (2\text{ units} \times 3\text{ units})$

$A_3 = 20\text{ square units} - 6\text{ square units}$

$A_3 = 14\text{ square units}$

I use composition and decomposition to calculate the areas of the different plots of land.

$A_4 = \frac{1}{2}(2\text{ units} \times 4\text{ units}) + (4\text{ units} \times 2\text{ units})$

$A_4 = 4\text{ square units} + 8\text{ square units}$

$A_4 = 12\text{ square units}$

The sum of the farmer's four pieces of land:

$A = 10\text{ square units} + 16\text{ square units} + 14\text{ square units} + 12\text{ square units}$
$A = 52\text{ square units}$

The sum of the farmer's four pieces of land in square feet:

4 units are 160 feet in length so 1 unit is 40 feet in length. Since each unit is square, each unit is 40 feet in length by 40 feet in width, or $1{,}600\text{ ft}^2$.

The total area of the farmer's pieces of land is $83{,}200\text{ ft}^2$ *because* $52 \times 1{,}600 = 83{,}200$.

The sum of the farmer's four pieces of land in acres:

$$83,200 \div 43,560 \approx 1.91$$

I can use the square feet to determine the number of acres the farmer owns. I use this information to determine the number of extra acres he purchases.

The farmer's four pieces of land total about 1.91 *acres.*

Extra land purchased with $\$5,000$*:*

$$3\text{ acres} - 1.91\text{ acres} = 1.09\text{ acres}$$

Extra land in square feet:

$$(1.09\text{ acres})\left(\frac{43,560\text{ ft}^2}{1\text{ acre}}\right) = 47,480.4\text{ ft}^2$$

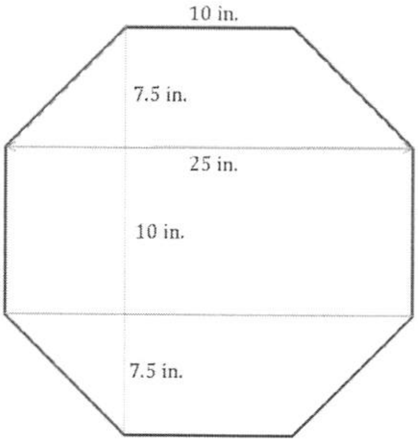
I need to determine the amount of extra square feet before I can determine the cost per square foot.

Price per square foot for extra land:

$$\frac{\$5,000}{47,480.4\text{ ft}^2} \approx \frac{\$0.11}{\text{ft}^2}$$

The farmer paid about $\$0.11$ *per square foot for the extra land.*

2. A stop sign is an octagon with eight equal sides and eight equal angles. The dimensions of the octagon are given below. One side of the octagon is to be painted red. If Derek has enough paint to cover 150 ft^2, can he paint 50 stop signs? Explain your answer.



I decompose the octagon into three quadrilaterals I am more familiar with.

Area of top trapezoid:

$$A = \frac{1}{2}(\mathbf{10\ in.} + \mathbf{25\ in.})(\mathbf{7.5\ in.})$$
$$A = \mathbf{131.25\ in^2}$$

Area of middle rectangle:

$$A = \mathbf{25\ in.} \times \mathbf{10\ in.}$$
$$A = \mathbf{250\ in^2}$$

Area of bottom trapezoid:

$$A = \frac{1}{2}(\mathbf{10\ in.} + \mathbf{25\ in.})(\mathbf{7.5\ in.})$$
$$A = \mathbf{131.25\ in^2}$$

Total area of a stop sign in square inches:

$$A = \mathbf{131.25\ in^2} + \mathbf{250\ in^2} + \mathbf{131.25\ in^2}$$
$$A = \mathbf{512.5\ in^2}$$

Total area of a stop sign in square feet:

I need to convert square inches to square feet to determine if Derek has enough paint for 50 stop signs.

$$\mathbf{512.5\ in^2}\left(\frac{\mathbf{1\ ft^2}}{\mathbf{144\ in^2}}\right) \approx \mathbf{3.56\ ft^2}$$

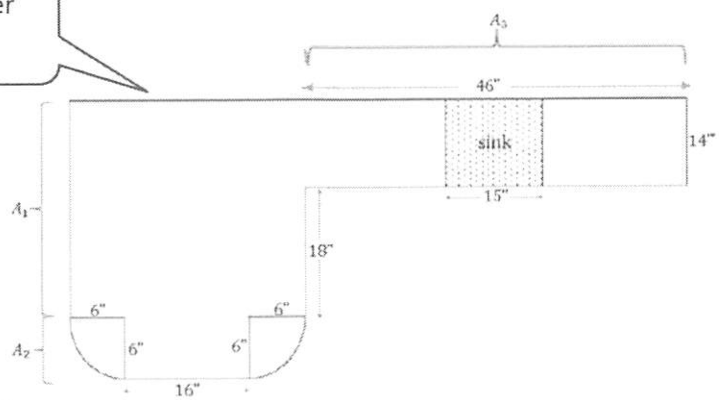
Total area of $\mathbf{50}$ ***stop signs:***

$$(\mathbf{3.56\ ft^2})(\mathbf{50}) = \mathbf{178\ ft^2}$$

Derek does not have enough paint for $\mathbf{50}$ ***stop signs because the total area of the stop signs is approximately*** $\mathbf{178\ ft^2}$***, which is more than*** $\mathbf{150\ ft^2}$***.***

3. A custom home builder is building a new kitchen. The diagram below is of a new kitchen countertop. Approximately how many square feet of counter space is there?

I decompose the counter top into three sections.



The width of the first section is 16 in. plus the radius (6 in.) of both quarter circles.

$$A_1 = (16 \text{ in.} + 12 \text{ in.})(14 \text{ in.} + 18 \text{ in.})$$
$$A_1 = (28 \text{ in.})(32 \text{ in.})$$
$$A_1 = 896 \text{ in}^2$$

The second section has a rectangle and two quarter circles. I remember the area formula for a quarter circle is $A = \frac{1}{4}\pi r^2$ and I can use 3.14 as an approximate value for π.

$$A_2 = (16 \text{ in.} \times 6 \text{ in.}) + \frac{1}{4}\pi(6 \text{ in.})^2 + \frac{1}{4}\pi(6 \text{ in.})^2$$
$$A_2 \approx 96 \text{ in}^2 + 28.26 \text{ in}^2 + 28.26 \text{ in}^2$$
$$A_2 \approx 152.52 \text{ in}^2$$

I subtract the area of the sink from the area of the third section because the countertop does not cover the sink.

$$A_3 = (46 \text{ in.} \times 14 \text{ in.}) - (15 \text{ in.} \times 14 \text{ in.})$$
$$A_3 = 644 \text{ in}^2 - 210 \text{ in}^2$$
$$A_3 = 434 \text{ in}^2$$

Total area of counter space in square inches:

$$A \approx 896 \text{ in}^2 + 152.52 \text{ in}^2 + 434 \text{ in}^2$$
$$A \approx 1{,}482.52 \text{ in}^2$$

Total area of counter space in square feet:

$$1{,}482.52 \text{ in}^2 \left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right) \approx 10.3 \text{ ft}^2$$

To answer the question, I need to convert in^2 to ft^2. I know 12 in. are in 1 ft., which means 144 in^2 are in 1 ft^2.

There is approximately 10.3 ft^2 of counter space.

G7-M6-Lesson 21: Mathematical Area Problems

1. In class, we generalized that $(a+b)^2 = a^2 + 2ab + b^2$. Use these results to evaluate the following expressions by writing $51 = 50 + 1$ and so on.
   a. Evaluate 51^2.

I can decompose 51 to the expression $50 + 1$. Therefore, 50 represents a in the general equation, and 1 represents b in the general equation.

$$\begin{aligned} \mathbf{51^2} &= \mathbf{(50+1)^2} \\ &= \mathbf{50^2 + 2(50 \cdot 1) + 1^2} \\ &= \mathbf{2{,}500 + 100 + 1} \\ &= \mathbf{2{,}601} \end{aligned}$$

   b. Evaluate 201^2.

$$\begin{aligned} \mathbf{201^2} &= \mathbf{(200+1)^2} \\ &= \mathbf{200^2 + 2(200 \cdot 1) + 1^2} \\ &= \mathbf{40{,}000 + 400 + 1} \\ &= \mathbf{40{,}401} \end{aligned}$$

I refer back to the general equation and substitute 200 for a and 1 for b. After substitution, I follow order of operations to simplify the expression.

   c. We can also generalize $(a-b) = a^2 - 2ab + b^2$. Use these results to evaluate the following expression by writing $99 = 100 - 1$, etc.
   Evaluate 99^2.

$$\begin{aligned} \mathbf{99^2} &= \mathbf{(100-1)^2} \\ &= \mathbf{100^2 - 2(100 \cdot 1) + 1^2} \\ &= \mathbf{10{,}000 - 200 + 1} \\ &= \mathbf{9{,}801} \end{aligned}$$

I recognize that I can decompose 99 into a subtraction expression. Therefore, I need to use the general rule for $(a-b)^2$ instead of $(a+b)^2$.

2. Use your knowledge that $a^2 - b^2 = (a-b)(a+b)$ to explain why:
   a. $40^2 - 10^2 = (30)(50)$.

   $\mathbf{40^2 - 10^2 = (40-10)(40+10) = (30)(50)}$

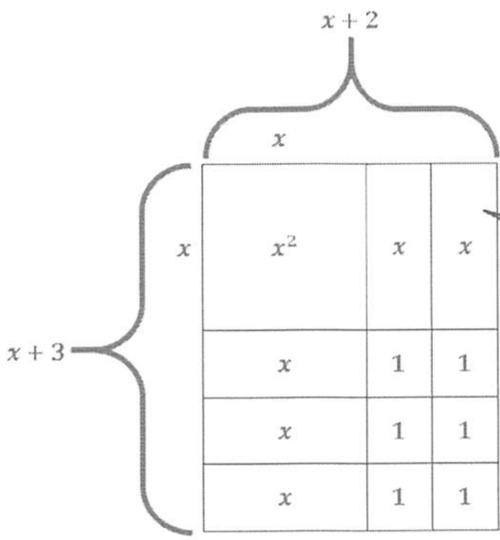
I can use the general equation to prove that the given number sentence is true. The first term, 40, represents a, and the second term, 10, represents b.

   b. $87^2 - 45^2 = (42)(132)$.

   $\mathbf{87^2 - 45^2 = (87-45)(87+45) = (42)(132)}$

3. Create a model for the product. Use the area model to write an equivalent expression that represents the area.

   $(x + 2)(x + 3)$



I remember creating these models multiple times during class.

$x^2 + 2x + 3x + 6$

4. Use the distributive property to multiply the following expressions.

   a. $(3 + 9)(4 + 8)$

$$\begin{aligned}(3+9)(4+8) &= (3+9)\cdot 4 + (3+9)\cdot 8\\ &= 3(4) + 9(4) + 3(8) + 9(8)\\ &= 12 + 36 + 24 + 72\\ &= 144\end{aligned}$$

I multiply each value in the first set of parentheses by each of the values in the second set of parentheses and then simplify.

   b. $(h - 5)(h + 5)$

$$\begin{aligned}(h-5)(h+5) &= (h-5)\cdot h + (h-5)\cdot 5\\ &= h(h) - 5(h) + h(5) - 5(5)\\ &= h^2 - 5h + 5h - 25\\ &= h^2 - 25\end{aligned}$$

There is a variable present in this expression, which means that I collect like terms.

I know $-5h$ and $5h$ are opposites, so their sum is 0. Therefore, there are only two terms in the product.

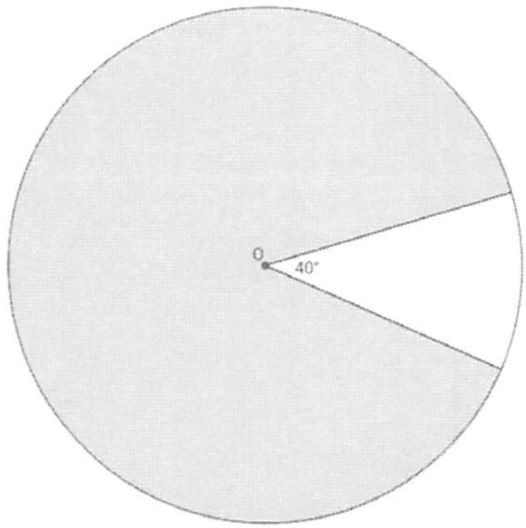
G7-M6-Lesson 22: Area Problems with Circular Regions

1. A circle with center O has an area of 117 in^2. Find the area of the unshaded region.

I know the entire circle represents $360°$, which means $40°$ represents $\frac{40}{360}$, or $\frac{1}{9}$, of the circle. Therefore, the area of the unshaded region is $\frac{1}{9}$ of the area of the entire circle.

$$A = \frac{1}{9}(117 \text{ in}^2) = 13 \text{ in}^2$$

The area of the unshaded region is 13 in^2.



2. The following region is bounded by the arcs of two quarter circles, each with a radius of 7 cm, and by line segments 10 cm in length. The region on the right shows a rectangle with dimensions 7 cm by 10 cm. Show that both shaded regions have equal areas.

Figure 1

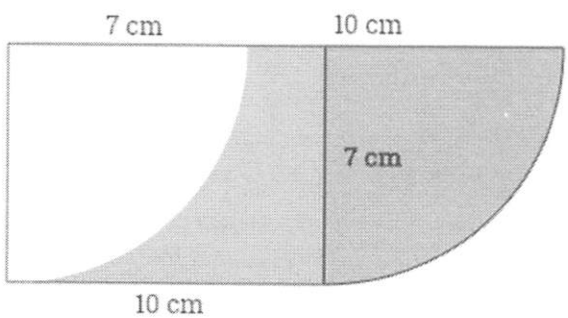
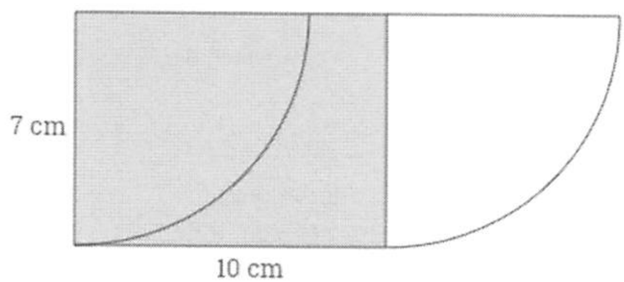


Figure 2



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For Figure 1, I subtract the area of the quarter circle from the area of the rectangle and then add the area of the extra quarter circle.

$A = \left(10 \text{ cm} \times 7 \text{ cm} - \frac{1}{4}\pi(7 \text{ cm})^2\right) + \frac{1}{4}\pi(7 \text{ cm})^2$

$A = 70 \text{ cm}^2 - \frac{49\pi}{4} \text{ cm}^2 + \frac{49\pi}{4} \text{ cm}^2$

$A = 70 \text{ cm}^2$

The shaded region in Figure 2 is a rectangle, which means I find the product of the length and width to calculate the area.

$A = 10 \text{ cm} \times 7 \text{ cm}$

$A = 70 \text{ cm}^2$

3. The diameters of four half circles are sides of a square with a side length of 6 cm.

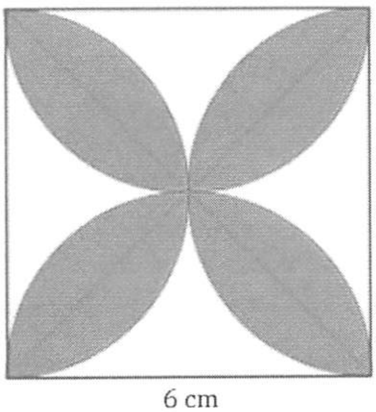


Figure 1

Figure 2 isolates one quarter of Figure 1.

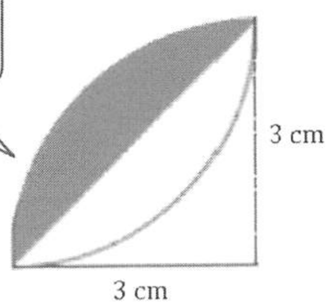


Figure 2
(Not drawn to scale)

a. Find the exact area of the shaded region.

Area of the shaded region in Figure 2:

$$\frac{1}{4}\pi(3 \text{ cm})^2 - \frac{1}{2}(3 \text{ cm} \times 3 \text{ cm})$$

$$\frac{9}{4}\pi \text{ cm}^2 - 4.5 \text{ cm}^2$$

To calculate the shaded region in Figure 2, I calculate the area of a quarter circle and subtract the area of a right triangle.

Total shaded area:

$$8\left(\frac{9}{4}\pi \text{ cm}^2 - 4.5 \text{ cm}^2\right)$$

$$18\pi \text{ cm}^2 - 36 \text{ cm}^2$$

There are 8 identical shaded regions, so I multiply the area by 8. To calculate the exact area, I leave π in the answer.

The exact area of the shaded region is $18\pi \text{ cm}^2 - 36 \text{ cm}^2$.

b. Find the approximate area using $\pi \approx \frac{22}{7}$.

To calculate the approximate area, I replace π in my expression from part (a) with $\frac{22}{7}$.

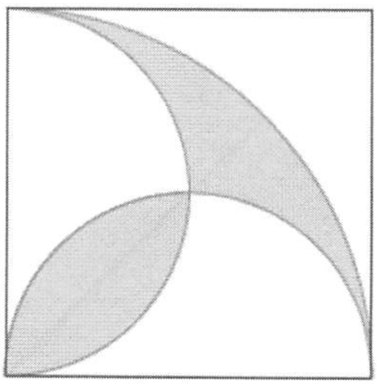
$$18\left(\frac{22}{7}\right)\ \text{cm}^2 - 36\ \text{cm}^2$$

$$56\frac{4}{7}\ \text{cm}^2 - 36\ \text{cm}^2$$

$$20\frac{4}{7}\ \text{cm}^2$$

The approximate area of the shaded region is $20\frac{4}{7}\ \text{cm}^2$.

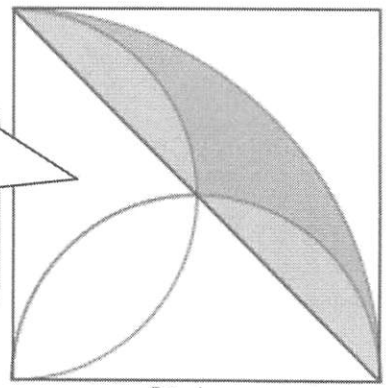
4. A square with a side length 8 inches is shown below, along with a quarter circle (with a side of the square as its radius) and two half circles (with diameters that are sides of the square). Write and explain a numerical expression that represents the exact area of the shaded region in the figure.



8 inches

Figure 1

Even though the shaded regions in both figures have the same area, it is easier to calculate the area of the shaded region in Figure 2.



8 inches

Figure 2

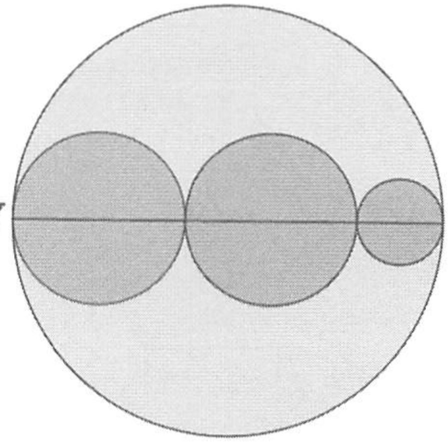
I recognize a quarter circle and a right triangle.

$$\frac{1}{4}\pi(8\ \text{in.})^2 - \left(\frac{1}{2}\cdot 8\ \text{in.}\cdot 8\ \text{in.}\right)$$

$$16\pi\ \text{in}^2 - 32\ \text{in}^2$$

The shaded area in each figure has the same area. This area can be found by subtracting the area of a right triangle with leg lengths of 8 in. *from the area of the quarter circle with a radius of* 8 in.

5. Four circles have centers on segment YZ. The diameters of the circles are in the ratio $5:2:2:1$. If the area of the largest circle is 100 ft^2, find the area inside the largest circle but outside the smaller circles.



The ratio of the areas of the four circles is $25:4:4:1$.

Let x represent the area of one of the medium circles in ft^2:

> I can use the ratio of the areas and the given area to determine the area of the medium circles.

$$\frac{4}{25}=\frac{x}{100}$$
$$(100)\left(\frac{4}{25}\right)=(100)\left(\frac{x}{100}\right)$$
$$16=x$$

Let y represent the area of the smaller circle in ft^2:

$$\frac{1}{25}=\frac{y}{100}$$
$$(100)\left(\frac{1}{25}\right)=(100)\left(\frac{y}{100}\right)$$
$$4=y$$

> Once I know the area of the three smaller circles, I can subtract these areas from the given area of the largest circle.

Area inside the largest circle but outside the smaller three circles:

$$A=100\text{ ft}^2-16\text{ ft}^2-16\text{ ft}^2-4\text{ ft}^2$$
$$A=64\text{ ft}^2$$

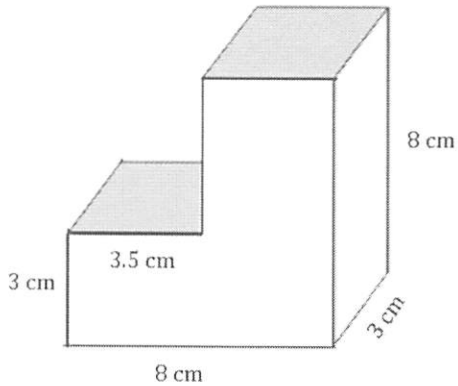
The area inside the largest circle but outside the three smaller circles is 64 ft^2.

G7-M6-Lesson 23: Surface Area

To calculate the surface area of a figure, I find the area of each face and then calculate the sum of these areas.

Determine the surface area of the figures.

1.



Even though the top and bottom of the figure look different, they cover the same area.

Area of the top and bottom: $2(8 \text{ cm} \times 3 \text{ cm}) = 48 \text{ cm}^2$

Area of left and right sides: $2(3 \text{ cm} \times 8 \text{ cm}) = 48 \text{ cm}^2$

Area of front and back:

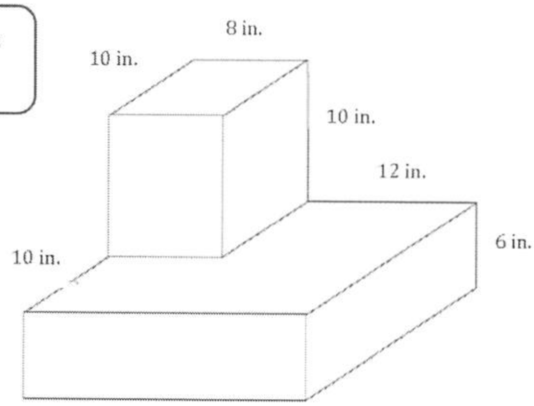
$2(8 \text{ cm} \times 3 \text{ cm}) + 2(4.5 \text{ cm} \times 5 \text{ cm}) = 93 \text{ cm}^2$

I decompose the front and back faces into two rectangles in order to calculate the area.

Total surface area: $48 \text{ cm}^2 + 48 \text{ cm}^2 + 93 \text{ cm}^2 = 189 \text{ cm}^2$

2.

The bottom of the top prism is not part of the surface area.



Surface area of top prism:

Area of top: $10 \text{ in.} \times 8 \text{ in.} = 80 \text{ in}^2$

Area of front and back sides: $2(8 \text{ in.} \times 10 \text{ in.}) = 160 \text{ in}^2$

Area of left and right sides: $2(10 \text{ in.} \times 10 \text{ in.}) = 200 \text{ in}^2$

Surface area of bottom prism:

Area of top: $(20 \text{ in.} \times 20 \text{ in.}) - 80 \text{ in}^2 = 320 \text{ in}^2$

Area of bottom: $20 \text{ in.} \times 20 \text{ in.} = 400 \text{ in}^2$

Area of front and back sides: $2(6 \text{ in.} \times 20 \text{ in.}) = 240 \text{ in}^2$

Area of left and right sides: $2(6 \text{ in.} \times 20 \text{ in.}) = 240 \text{ in}^2$

I subtract 80 in^2 from the area of the top of the bottom prism because this is the area that the top and bottom prisms overlap.

Total surface area: $80 \text{ in}^2 + 160 \text{ in}^2 + 200 \text{ in}^2 + 320 \text{ in}^2 + 400 \text{ in}^2 + 240 \text{ in}^2 + 240 \text{ in}^2 = 1,640 \text{ in}^2$

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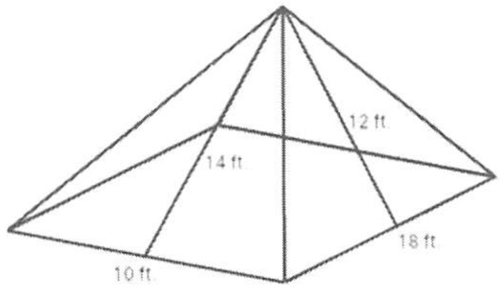
3. I know that the prism has a rectangular base and four triangular faces. I remember that the area formula for a triangle is $A = \frac{1}{2}bh$.



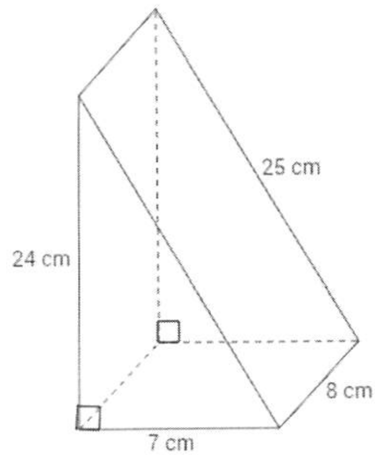
Area of front and back: $2\left(\frac{1}{2}(10 \text{ ft.} \times 14 \text{ ft.})\right) = 140 \text{ ft}^2$

Area of left and right: $2\left(\frac{1}{2}(18 \text{ ft.} \times 12 \text{ ft.})\right) = 216 \text{ ft}^2$

Area of the base: $10 \text{ ft.} \times 18 \text{ ft.} = 180 \text{ ft}^2$

Total surface area: $140 \text{ ft}^2 + 216 \text{ ft}^2 + 180 \text{ ft}^2 = 536 \text{ ft}^2$

4. I know the triangular prism has two triangular bases and three rectangular faces. I need to find the area of each of these five faces in order to calculate the surface area.



Area of the front and back:

$$2\left(\frac{1}{2}(7 \text{ cm} \times 24 \text{ cm})\right) = 168 \text{ cm}^2$$

Area of bottom:

$$7 \text{ cm} \times 8 \text{ cm} = 56 \text{ cm}^2$$

Area that can be seen from left side:

$$24 \text{ cm} \times 8 \text{ cm} = 192 \text{ cm}^2$$

Area that can be seen from the right side:

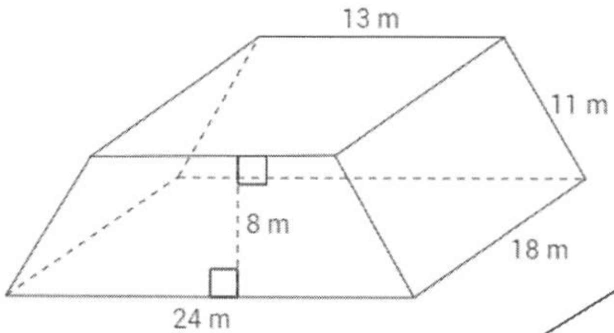
$$25 \text{ cm} \times 8 \text{ cm} = 200 \text{ cm}^2$$

Total surface area: $168 \text{ cm}^2 + 56 \text{ cm}^2 + 192 \text{ cm}^2 + 200 \text{ cm}^2 = 616 \text{ cm}^2$

G7-M6-Lesson 24: Surface Area

Determine the surface area of each figure.

1.



I know that trapezoidal prisms have two bases that are trapezoids. The area formula for a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$, where b_1 and b_2 represent the lengths of the two bases.

Area of front and back: $2\left(\frac{1}{2}(13\text{ m} + 24\text{ m})8\text{ m}\right) = 296\text{ m}^2$

Area of top: $13\text{ m} \times 18\text{ m} = 234\text{ m}^2$

Area of left and right sides: $2(11\text{ m} \times 18\text{ m}) = 396\text{ m}^2$

Area of bottom: $24\text{ m} \times 18\text{ m} = 432\text{ m}^2$

The other four faces are all rectangles.

Total surface area: $296\text{ m}^2 + 234\text{ m}^2 + 396\text{ m}^2 + 432\text{ m}^2 = 1,358\text{ m}^2$

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2. Determine the surface area after two square holes with a side length of 3 m are cut through the solid figure composed of two rectangular prisms.

Surface area of the top prism before the hole is cut:

Area of top: $5\text{ m} \times 6\text{ m} = 30\text{ m}^2$

Area of front and back: $2(5\text{ m} \times 6\text{ m}) = 60\text{ m}^2$

Area of sides: $2(6\text{ m} \times 6\text{ m}) = 72\text{ m}^2$

I first find the surface area of both prisms before the holes are cut.

Surface area of the bottom prism before the hole is cut:

Area of top: $12\text{ m} \times 12\text{ m} - 30\text{ m}^2 = 114\text{ m}^2$

Area of bottom: $12\text{ m} \times 12\text{ m} = 144\text{ m}^2$

Area of front and back: $2(12\text{ m} \times 4\text{ m}) = 96\text{ m}^2$

Area of sides: $2(12\text{ m} \times 4\text{ m}) = 96\text{ m}^2$

Each of the four faces inside each hole are rectangles. The width of all eight rectangles is 3 m. The length of the rectangles in the top hole is 5 m. The length of the rectangles in the bottom hole is the same as the height of the bottom prism, or 4 m.

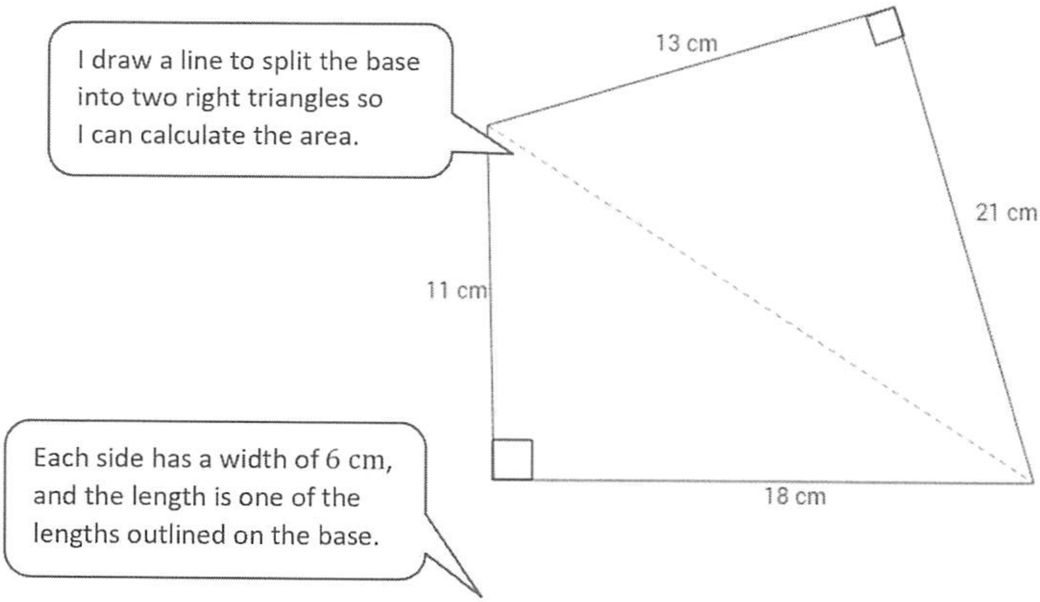
Surface area of holes: $4(3\text{ m} \times 5\text{ m}) + 4(3\text{ m} \times 4\text{ m}) = 108\text{ m}^2$

I add all the areas together, but I also have to subtract the places where holes are cut. Each hole is a 3 m by 3 m square, which means the area cut is 9 m^2. There are four places this area is cut out of the original prism, which means I subtract an area of 36 m^2 from the total surface area.

Total surface area:

$30\text{ m}^2 + 60\text{ m}^2 + 72\text{ m}^2 + 114\text{ m}^2 + 144\text{ m}^2 + 96\text{ m}^2 + 96\text{ m}^2 + 108\text{ m}^2 - 36\text{ m}^2 = 684\text{ m}^2$

3. The base of a right prism is shown below. Determine the surface area if the height of the prism is 6 cm. Explain how you determined the surface area.



Area of sides: $(11\text{ cm}\times 6\text{ cm})+(18\text{ cm}\times 6\text{ cm})+(21\text{ cm}\times 6\text{ cm})+(13\text{ cm}\times 6\text{ cm})=378\text{ cm}^2$

Area of bases: $2\left(\frac{1}{2}(11\text{ cm}\times 18\text{ cm})\right)+2\left(\frac{1}{2}(21\text{ cm}\times 13\text{ cm})\right)=471\text{ cm}^2$

Total surface area: $471\text{ cm}^2+378\text{ cm}^2=849\text{ cm}^2$

G7-M6-Lesson 25: Volume of Right Prisms

1. Two right prism containers each hold 37.5 gallons of water. The height of the first container is 12 inches. The height of the second container is 10 inches. If the area of the base of the first container is $5\ \text{ft}^2$, find the area of the base of the second container. Explain your reasoning.

Let B represent the area of the base in the second container.

$$12 \times 5 = 10 \times B$$
$$60 = 10B$$
$$\left(\frac{1}{10}\right)(60) = \left(\frac{1}{10}\right)(10B)$$
$$6 = B$$

I know the two containers have the same volume. Therefore, the product of the area of the base and the height of each container must be equal.

The area of the base of the second container is $6\ \text{ft}^2$.

There are three different dimensions in a right rectangular prism: length, width, and height.

2. Two containers are shaped like right rectangular prisms. Each of the larger container's dimensions are 30% more than the smaller container's dimensions. If the smaller container holds 15 gallons when full, how many gallons does the larger container hold? Explain your reasoning.

Each dimension of the larger container is 1.3 times larger than those of the smaller container because $100\% + 30\%$ is 130%.

The volume of the larger container is 1.3^3, or 2.197, times larger than the volume of the smaller container.

$$15 \text{ gallons} \times 2.197 = 32.955 \text{ gallons}$$

The volume of the larger container is 2.197 times bigger than the volume of the smaller container.

The volume of the larger container is 32.955 gallons.

3. An aquarium in the shape of a right rectangular prism has a base area of 40 in^2 and height of 13 in. Currently, the aquarium is only partially filled, and the height of the water is 8 in. A few decorations are added to the bottom of the aquarium, which makes the water rise to the top, completely submerging the decorations but without causing overflow. Find the volume of the decorations.

The height of the water increased 5 in. because the height increased from 8 in. to 13 in.

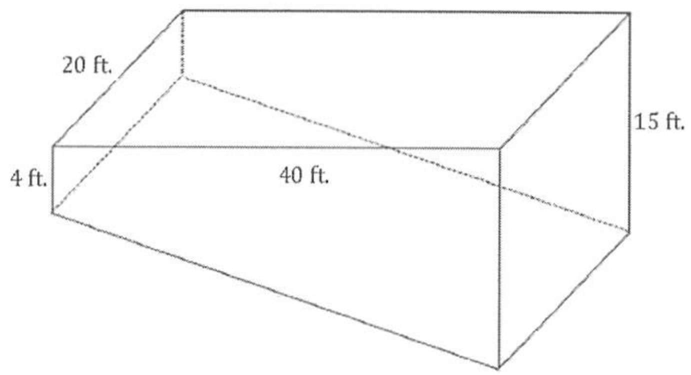
$$40 \text{ in}^2 \times 5 \text{ in.} = 200 \text{ in}^3$$

The volume of the decorations is 200 in^3.

The volume of the decorations will be the same as the change in the volume of the water. I can calculate the change in the volume by multiplying the change in height by the area of the base.

4. A rectangular swimming pool is 20 feet wide and 40 feet long. The pool is 4 feet deep at one end, and 15 feet deep at the other.

   a. Sketch the swimming pool as a right prism.



   b. What kind of prism is the swimming pool?

   The swimming pool is a right trapezoidal prism.

There are two trapezoidal bases in the swimming pool.

c. What is the volume of the swimming pool in cubic feet?

The area of the trapezoidal base is half the sum of the lengths times the height.

$$V = \left(\frac{1}{2}(4\text{ ft.} + 15\text{ ft.})(40\text{ ft.})\right)(20\text{ ft.})$$

$$V = \left(\frac{1}{2}(19\text{ ft.})(40\text{ ft.})\right)(20\text{ ft.})$$

$$V = (380\text{ ft}^2)(20\text{ ft.})$$

$$V = 7{,}600\text{ ft}^3$$

The volume of the swimming pool is $7{,}600\text{ ft}^3$.

d. How many gallons will the swimming pool hold if each cubic feet of water is about 7.5 gallons?

$$(7{,}600\text{ ft}^3)\left(\frac{7.5\text{ gallons}}{1\text{ ft}^3}\right) = 57{,}000\text{ gallons}$$

The swimming pool will hold about $57{,}000$ *gallons of water.*

G7-M6-Lesson 26: Volume of Composite Three-Dimensional Objects

1. Find the volume of the three-dimensional object composed of right rectangular prisms.

Volume of top and bottom prisms:

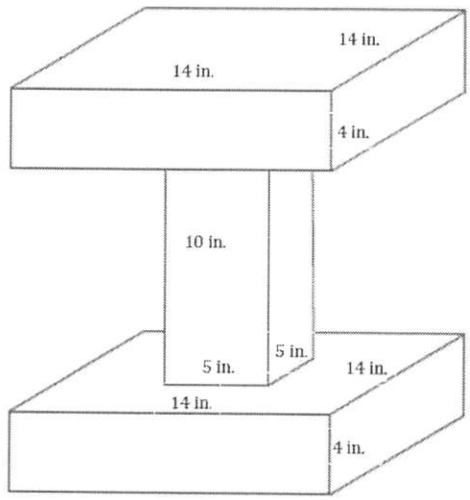
$2(\mathbf{14\ in.} \times \mathbf{14\ in.} \times \mathbf{4\ in.}) = \mathbf{1,568\ in^3}$

Volume of middle prism:

$\mathbf{5\ in.} \times \mathbf{5\ in.} \times \mathbf{10\ in.} = \mathbf{250\ in^3}$

Total volume:

$\mathbf{1,568\ in^3} + \mathbf{250\ in^3} = \mathbf{1,818\ in^3}$



Similar to surface area, I can find the volume of each part of the object and then calculate the total volume by finding the sum of all the partial volumes.

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2. Two students are finding the volume of a prism with a rhombus base but are provided different information regarding the prism. One student receives Figure 1 while the other receives Figure 2.

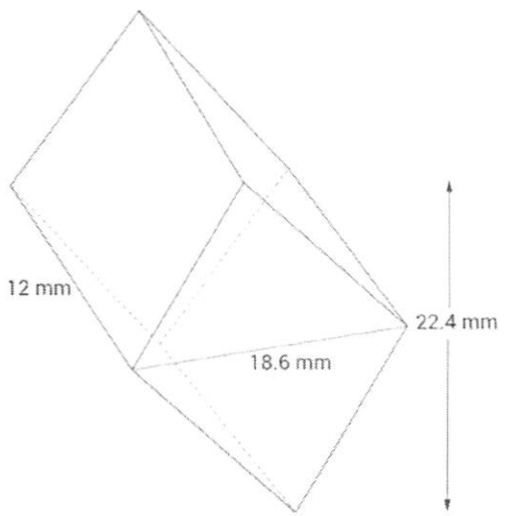


Figure 1

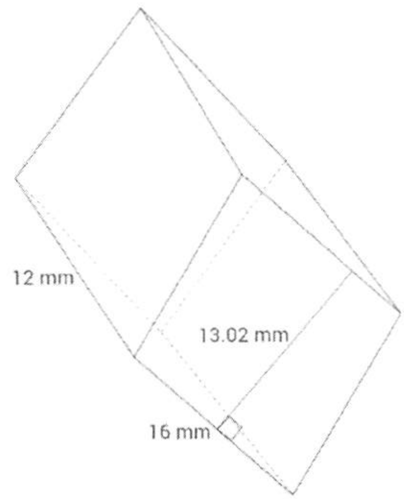


Figure 2

Find the volume in each case; show that the volumes are equal.

In Figure 1, I split the base into two triangles in order to find the area of the base, and then I multiply by the height of the prism.

Volume of Figure 1: $2\left(\frac{1}{2}(18.6\text{ mm} \times 11.2\text{ mm})\right) \times 12\text{ mm} = 2,499.84\text{ mm}^3$

Volume of Figure 2: $16\text{ mm} \times 13.02\text{ mm} \times 12\text{ mm} = 2,499.84\text{ mm}^3$

In Figure 2, I can use the information provided to find the area of the base and then multiply by the height of the prism.

3. A plastic die cube for a game has an edge length of 2.5 cm. Throughout the cube, there are 15 cubic cutouts, each with an edge length of 3 mm. What is the volume of the cube?

I know all sides of a cube are equal length.

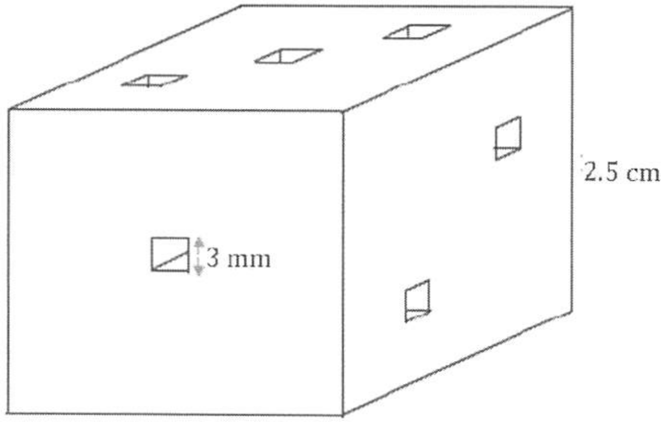
Volume of large cube:

$$(2.5\text{ cm})^3 = 15.625\text{ cm}^3$$

Volume of cutout cubes:

$$15(3\text{ mm})^3 = 405\text{ mm}^3$$

I notice that the units for the two different volumes don't match. I convert mm^3 to cm^3 by dividing by 1,000.



Total volume of the die: $15.625\text{ cm}^3 - 0.405\text{ cm}^3 = 15.22\text{ cm}^3$

I find the total volume by subtracting the volume of the cutouts from the volume of the large cube.

4. A right rectangular prism has each of its dimensions (length, width, and height) increased by 20%. By what percent is its volume increased?

When I increase each dimension by 20%, the new dimensions will be 100% + 20%, or 120%, of the original dimension.

$$V' = 1.2l \cdot 1.2w \cdot 1.2h$$
$$V' = 1.728lwh$$

The larger volume is 172.8% *of the smaller volume, which means the volume increased by* 72.8%.

The smaller volume represents 100%, so the increase is the difference between the two percentages.

G7-M6-Lesson 27: Real-World Volume Problems

1. Olivia has a leak in her new roof, so she puts a container in the shape of a right rectangular prism under the leak. Rainwater is dripping into the container at an average rate of 14 drops per minute. The container Olivia places under the leak has dimensions of $6\text{ cm} \times 4\text{ cm} \times 9\text{ cm}$. Assuming each rain drop is roughly 1 cm^3, approximately how long does Olivia have before the container overflows?

   Volume of the container:

   $$6\text{ cm} \times 4\text{ cm} \times 9\text{ cm} = 216\text{ cm}^3$$

   Number of minutes until the container is filled with rainwater:

   $$216\text{ cm}^3\left(\frac{1\text{ min.}}{14\text{ cm}^3}\right) \approx 15.43\text{ min.}$$

   The bucket will overflow in about 15.43 minutes.

   I determine the volume of the container and then use the rate to determine how long Olivia has until the container overflows.

2. A basement flooded and contains $10{,}000\text{ ft}^3$ of water that needs to be drained. At 1:00 p.m., a pump is turned on that drains water at the rate of 9 ft^3 per minute. Four hours later, at 5:00 p.m., a second pump is activated that drains water at the rate of 5 ft^3 per minute. At what time will the basement be free of water?

   Water drained during the first four hours:

   $$\left(\frac{9\text{ ft}^3}{1\text{ min.}}\right)(240\text{ min.}) = 2{,}160\text{ ft}^3$$

   Volume of water that still needs to be drained:

   $$10{,}000\text{ ft}^3 - 2{,}160\text{ ft}^3 = 7{,}840\text{ ft}^3$$

   Once I determine how much water is drained when only one pump is working, I can determine how much water is left in the basement.

   Amount of time needed to drain the remaining water:

   When both pumps are working, I know the rate is 14 ft^3 per minute because I add the two rates together.

   $$(7{,}840\text{ ft}^3)\left(\frac{1\text{ min.}}{14\text{ ft}^3}\right) = 560\text{ min.}$$

   It will take $13\frac{1}{3}$ hours to drain the basement, which means the basement will be free of water at 2:20 a.m.

3. A pool contains 12,000 ft^3 of water. Pump A can drain the pool in 10 hours, and Pump B can drain the pool in 15 hours. How long will it take both pumps working together to drain the pool?

Rate at which Pump A drains the pool: $\frac{1}{10}$ *pool per hour*

Rate at which Pump B drains the pool: $\frac{1}{15}$ *pool per hour*

I can determine the rate at which each pump drains the pool in order to determine the rate the water drains when both pumps are working together.

I remember to find common denominators before adding the fractions.

Together, the pumps drain the pool at $\left(\frac{1}{10}+\frac{1}{15}\right)$ *pool per hour, or* $\frac{1}{6}$ *pool per hour. Therefore, it will take* 6 *hours to drain the pool when both pumps are working together.*

4. A 1,500-gallon aquarium can be filled with water flowing at a constant rate in 6 hours. When a decorative rock is placed in the aquarium, it can be filled in 5.25 hours. Find the volume of the rock in cubic feet ($1\text{ ft}^3 = 7.48$ gal.).

Rate of the water flow into aquarium:

$$\frac{\mathbf{1{,}500\ gal.}}{\mathbf{6\ hours}} = \frac{\mathbf{250\ gal.}}{\mathbf{1\ hour}}$$

Volume of the rock in gallons:

$$\left(\frac{\mathbf{250\ gal.}}{\mathbf{1\ hour}}\right)(\mathbf{0.75\ hour}) = \mathbf{187.5\ gal.}$$

When the rock is placed in the aquarium, I know that it takes 0.75 hours less to fill. I can use the unit rate and time to determine the volume of the rock.

Volume of the rock in cubic feet:

$$(\mathbf{187.5\ gal.})\left(\frac{\mathbf{1\ ft^3}}{\mathbf{7.48\ gal.}}\right) \approx \mathbf{25.07\ ft^3}$$

The volume of the rock is approximately $\mathbf{25.07\ ft^3}$.

In order to answer the question, I need to convert the volume of the rock from gallons to cubic feet.