Homework Helpers

Grade 8
Module 4

G8-M4-Lesson 1: Writing Equations Using Symbols

Write each of the following statements using symbolic language.

1. George is four years older than his sister Sylvia.
George's other sister is five years younger than Sylvia.
The sum of all of their ages is 68 years.

   Let x be Sylvia's age. Then,
   $(x+4)+(x-5)+x=68.$

   > Since I know something about Sylvia and both her brother and sister, I will define my variable as Sylvia's age.

2. The sum of three consecutive integers is 843.

   Let x be the first integer. Then,
   $x+(x+1)+(x+2)=843.$

   > I remember that consecutive means one after the next. If my first number was 5, then a numeric statement would look like this:
   > $5+(5+1)+(5+2)$.
   > I need to write something similar using symbols.

3. One number is two more than another number.
The sum of their squares is 33.

   Let x be the smaller number. Then,
   $x^2+(x+2)^2=33.$

4. When you add 42 to $\frac{1}{3}$ of a number, you get the number itself.

   Let x be the number. Then,
   $\frac{1}{3}x+42=x.$

   > I don't know what number a fraction of 45 is. I remember that taken away from 23 means I need to subtract the number from 23.

5. When a fraction of 45 is taken away from 23, what remains exceeds one-half of eleven by twelve.

   Let x be the fraction of 45. Then,
   $23-x=\frac{1}{2}\cdot 11+12.$

6. The sum of three consecutive odd integers is 165. Let x be the middle of the three odd integers. Transcribe the statement accordingly.

   $(x-2)+x+(x+2)=165$

   > If the middle number is odd, then I need to subtract two to get the odd integer before it, and I need to add two to get the odd integer after it.

G8-M4-Lesson 2: Linear and Non-Linear Expressions in x

Write each of the following statements as a mathematical expression. State whether the expression is linear or nonlinear. If it is nonlinear, then explain why.

1. A number added to five cubed

   Let x be a number; then, $5^3 + x$ is a linear expression.

   It is linear because it a sum of constants and x to the 1st power.

2. The quotient of seven and a number, added to twenty-five

   Let x be a number; then, $\frac{7}{x} + 25$ is a nonlinear expression.
   The term $\frac{7}{x}$ is the same as $7 \cdot \frac{1}{x}$ and $\frac{1}{x} = x^{-1}$, which is why it is not linear.

   I remember that $\frac{1}{x} = x^{-1}$ from the beginning of the year.

3. The sum that represents the number of hotdogs sold if 148 hotdogs were sold Thursday, half of the remaining hotdogs were sold on Friday, and 203 hotdogs were sold on Saturday

   Let x be the remaining number of hotdogs; then, $148 + \frac{1}{2}x + 203$ is a linear expression.

4. The product of 46 and a number, added to the reciprocal of the number squared

   Let x be a number; then, $46x + \frac{1}{x^2}$ is a nonlinear expression.
   The term $\frac{1}{x^2}$ is the same as x^{-2}, which is why it is not linear.

   I could write the expression as $\frac{1}{x^2} + 46x$ by applying the commutative property of addition.

5. The product of 12 and a number and then the product multiplied by itself seven times

   Let x be a number; then, $(12x)^7$ is a nonlinear expression. The expression can be written as $12^7 \cdot x^7$. The exponent of 7 with a base of x is the reason it is not linear.

6. The sum of seven and a number, multiplied by the number

   Let x be a number; then, $(7 + x)x$ is a nonlinear expression because $(7 + x)x = 7x + x^2$ after using the distributive property. It is nonlinear because the power of x in the term x^2 is greater than 1.

   I need to use parentheses around the sum of seven and a number.

G8-M4-Lesson 3: Linear Equations in x

1. Given that $5x - 3 = 17$ and $7x + 3 = 17$, does $5x - 3 = 7x + 3$? Explain.

   Yes, $5x - 3 = 7x + 3$ because a linear equation is a statement about equality. We are given that $5x - 3$ is equal to 17, but $7x + 3$ is also equal to 17. Since each linear expression is equal to the same number, the expressions are equal, $5x - 3 = 7x + 3$.

Since the left side of both expressions are equal to the same number, I can say that the expressions are equal to each other.

2. Is 5 a solution to the equation $3x - 1 = 5x + 7$? Explain.

   If we replace x with the number 5, then the left side of the equation is

$$\begin{aligned} 3 \cdot (5) - 1 &= 15 - 1 \\ &= 14, \end{aligned}$$

   and the right side of the equation is

$$\begin{aligned} 5 \cdot (5) + 7 &= 25 + 7 \\ &= 32. \end{aligned}$$

   Since $14 \neq 32$, 5 is not a solution of the equation $3x - 1 = 5x + 7$.

I need to see if the right side is equal to the left side when I replace x with the number 5. If the left side is not equal to the right side, then I know 5 is not a solution.

I know that a linear equation is really a question that is asking what number x will satisfy the equation.

3. Use the linear equation $11(x - 2) = 11x - 22$ to answer parts (a)–(c).

   a. Does $x = 3$ satisfy the equation above? Explain.

   If we replace x with the number 3, then the left side of the equation is

$$\begin{aligned} 11(x - 2) &= 11(3 - 2) \\ &= 11(1) \\ &= 11, \end{aligned}$$

   and the right side of the equation is

$$\begin{aligned} 11x - 22 &= 11 \cdot 3 - 22 \\ &= 33 - 22 \\ &= 11. \end{aligned}$$

   Since $11 = 11$, then $x = 3$ is a solution of the equation $11(x - 2) = 11x - 22$.

b. Is $x = -\frac{1}{2}$ a solution of the equation above? Explain.

If we replace x with the number $-\frac{1}{2}$, then the left side of the equation is

$$\begin{aligned} 11(x-2) &= 11\left(-\frac{1}{2}-2\right) \\ &= 11\left(-\frac{1}{2}-\frac{4}{2}\right) \\ &= 11\left(-\frac{5}{2}\right) \\ &= -\frac{55}{2}, \end{aligned}$$

I can rewrite 2 as an equivalent fraction with the same denominator as $\frac{1}{2}$.

and the right side of the equation is

$$\begin{aligned} 11x-22 &= 11\cdot-\frac{1}{2}-22 \\ &= -\frac{11}{2}-22 \\ &= -\frac{11}{2}-\frac{44}{2} \\ &= -\frac{55}{2}. \end{aligned}$$

Since the right side is equal to the left side, $-\frac{1}{2}$ is a solution.

Since $-\frac{55}{2} = -\frac{55}{2}$, $x = -\frac{1}{2}$ is a solution of the equation $11(x-2) = 11x-22$.

c. What interesting fact about the equation $11(x-2) = 11x-22$ is illuminated by the answers to parts (a) and (b)? Why do you think this is true?

I notice that the equation $11(x-2) = 11x-22$ is an identity under the distributive law.

I remember my teacher saying illuminated means "What do I notice?"

I think I can choose any number for x and the equation will be true.

G8-M4-Lesson 4: Solving a Linear Equation

For each problem, show your work, and check that your solution is correct.

1. Solve the linear equation $5x - 7 + 2x = -21$. State the property that justifies your first step and why you chose it.

   I used the commutative and distributive properties on the left side of the equal sign to simplify the expression to fewer terms.

$$\begin{aligned} 5x - 7 + 2x &= -21 \\ 5x + 2x - 7 &= -21 \\ (5+2)x - 7 &= -21 \\ 7x - 7 &= -21 \\ 7x - 7 + 7 &= -21 + 7 \\ 7x &= -14 \\ \frac{1}{7}(7)x &= \frac{1}{7}(-14) \\ x &= -2 \end{aligned}$$

The commutative property allows me to rearrange and group terms within expressions. The distributive property allows me to simplify expressions by combining terms that are alike.

Check: The left side is equal to $5(-2) - 7 + 2(-2) = -10 - 7 - 4 = -21$, which is equal to the right side. Therefore, $x = -2$ is a solution to the equation $5x - 7 + 2x = -21$.

2. Solve the linear equation $\frac{1}{7}x - 11 = \frac{1}{4}x - 14$. State the property that justifies your first step and why you chose it.

I chose to use the addition property of equality to get all of the constants on one side of the equal sign and the subtraction property of equality to get all of the terms with an x on the other side of the equal sign.

$$\frac{1}{7}x - 11 = \frac{1}{4}x - 14$$
$$\frac{1}{7}x - 11 + 11 = \frac{1}{4}x - 14 + 11$$
$$\frac{1}{7}x - \frac{1}{4}x = \frac{1}{4}x - \frac{1}{4}x - 3$$
$$\left(\frac{1}{7} - \frac{1}{4}\right)x = -3$$
$$\left(\frac{4}{28} - \frac{7}{28}\right)x = -3$$
$$-\frac{3}{28}x = -3$$
$$-\frac{28}{3}\left(-\frac{3}{28}\right)x = -\frac{28}{3}(-3)$$
$$x = 28$$

I remember that the order doesn't matter, as long as I use the properties of equality correctly. I could use the subtraction property of equality to get all the terms with an x on one side of the equal sign and then use the addition property of equality to get all the constants on the other side.

Check: The left side of the equation is $\frac{1}{7}(28) - 11 = 4 - 11 = -7$. The right side of the equation is $\frac{1}{4}(28) - 14 = 7 - 14 = -7$. Since both sides equal -7, $x = 28$ is a solution to the equation $\frac{1}{7}x - 11 = \frac{1}{4}x - 14$.

I need to check my answer in the original equation because I may have made a mistake when transforming the equation.

3. Corey solved the linear equation $5x + 7 - 18x = 14 + 3x - 87$. His work is shown below. When he checked his answer, the left side of the equation did not equal the right side. Find and explain Corey's error, and then solve the equation correctly.

$$5x + 7 - 18x = 14 + 3x - 87$$
$$-13x + 7 = 3x - 73$$
$$-13x + 7 + 3x = 3x - 73 - 3x$$
$$-10x + 7 = -73$$
$$-10x + 7 - 7 = -73 - 7$$
$$-10x = -80$$
$$\frac{-10}{-10}x = \frac{-80}{-10}$$
$$x = 8$$

A strategy I used in class is to solve the linear equation and check my answer without looking at Corey's solution. I will compare my solution to Corey's to see if I find any differences.

Corey made a mistake on the third line. He added $3x$ to the left side of the equal sign and subtracted $3x$ on the right side of the equal sign. To use the property correctly, he should have subtracted $3x$ on both sides of the equal sign, making the equation at that point:

$$\boldsymbol{-13x + 7 - 3x = 3x - 73 - 3x}$$
$$\boldsymbol{-16x + 7 = -73}$$
$$\boldsymbol{-16x + 7 - 7 = -73 - 7}$$
$$\boldsymbol{-16x = -80}$$
$$\boldsymbol{\frac{-16}{-16}x = \frac{-80}{-16}}$$
$$\boldsymbol{x = 5}$$

G8-M4-Lesson 5: Writing and Solving Linear Equations

For each of the following problems, write an equation and solve.

The sum of the measures of complementary angles is $90°$.

1. An angle measures eleven more than four times a number. Its complement is two more than three times the number. What is the measure of each angle in degrees?

   Let x be the number. Then, the measure of one angle is $4x + 11$. The measure of the other angle is $3x + 2$. Since the angles are complementary, the sum of their measures will be $90°$.

$$
\begin{aligned}
4x + 11 + 3x + 2 &= 90 \\
7x + 13 &= 90 \\
7x + 13 - 13 &= 90 - 13 \\
7x &= 77 \\
x &= 11
\end{aligned}
$$

I'm not done yet. I need to make sure I find the measure of each angle.

   Replacing x with 11 in $4x + 11$ gives $4(11) + 11 = 44 + 11 = 55$.
   Replacing x with 11 in $3x + 2$ gives $3(11) + 2 = 33 + 2 = 35$.
   Therefore, the measures of the angles are $55°$ and $35°$.

2. The angles of a triangle are described as follows: $\angle A$ is the smallest angle. The measure of $\angle B$ is one more than the measure of $\angle A$. The measure of $\angle C$ is 3 more than twice the measure of $\angle A$. Find the measures of the three angles in degrees.

   Let x be the measure of $\angle A$. Then, the measure of $\angle B$ is $x + 1°$ and $\angle C$ is $2x + 3°$. The sum of the measures of the angles must be $180°$.

$$
\begin{aligned}
x + x + 1° + 2x + 3° &= 180° \\
4x + 4° &= 180° \\
4x + 4° - 4° &= 180° - 4° \\
4x &= 176° \\
x &= 44°
\end{aligned}
$$

The sum of the measures of the interior angles of a triangle is $180°$.

   The measures of the angles are as follows: $\angle A = 44°$, $\angle B = 45°$, and $\angle C = 2(44°) + 3° = 88° + 3° = 91°$.

3. A pair of corresponding angles are described as follows: The measure of one angle is fifteen less than four times a number, and the measure of the other angle is twenty more than four times the number. Are the angles congruent? Why or why not?

I need to use the fact that corresponding angles of parallel lines are congruent so that I can write an equation.

Let x be the number. Then, the measure of one angle is $4x - 15$, and the measure of the other angle is $4x + 20$. Assume they are congruent, which means their measures are equal.

$$4x - 15 = 4x + 20$$
$$4x - 4x - 15 = 4x - 4x + 20$$
$$-15 \neq 20$$

Since $-15 \neq 20$, the angles are not congruent.

4. Three angles are described as follows: $\angle A$ is one-third the size of $\angle B$. The measure of $\angle C$ is equal to seven more than three times the measure of $\angle B$. The sum of the measures of $\angle A$ and $\angle C$ is $147°$. Can the three angles form a triangle? Why or why not?

Since I don't know if the three angle measures form a triangle, I need to use the sum of the two triangles to write my equation.

Let x represent the measure of $\angle B$. Then, the measure of $\angle A$ is $\frac{x}{3}$ and the measure of $\angle C$ is $3x + 7°$.

The sum of the measures of $\angle A$ and $\angle C$ is $147°$.

$$\frac{x}{3} + 3x + 7° = 147°$$
$$\frac{1}{3}x + \frac{9}{3}x + 7° = 147°$$
$$\left(\frac{1}{3} + \frac{9}{3}\right)x + 7° = 147°$$
$$\frac{10}{3}x + 7° - 7° = 147° - 7°$$
$$\frac{10}{3}x = 140°$$
$$10x = 420°$$
$$x = 42°$$

I need to check the sum of the three angles to see if they form a triangle.

The measure of $\angle A$ is $\left(\frac{42}{3}\right)^{\circ} = 14°$, the measure of $\angle B$ is $42°$, and the measure of $\angle C$ is $3(42°) + 7° = 133°$. The sum of the three angles is $14° + 42° + 133° = 189°$. Since the sum of the measures of the interior angles of a triangle must have a sum of $180°$, these angles do not form a triangle. Their sum is too large.

G8-M4-Lesson 6: Solutions of a Linear Equation

Transform the equation if necessary, and then solve it to find the value of x that makes the equation true.

1. $3x - (x + 2) + 11x = \frac{1}{2}(4x - 8)$

The negative sign in front of the parentheses means to take the opposite of each term inside the parentheses.

$$\begin{aligned}
3x - (x + 2) + 11x &= \frac{1}{2}(4x - 8) \\
3x - x - 2 + 11x &= 2x - 4 \\
13x - 2 &= 2x - 4 \\
13x - 2x - 2 &= 2x - 2x - 4 \\
11x - 2 &= -4 \\
11x - 2 + 2 &= -4 + 2 \\
11x &= -2 \\
\frac{11}{11}x &= -\frac{2}{11} \\
x &= -\frac{2}{11}
\end{aligned}$$

I need to use the distributive property to each term inside the parentheses. It will allow me to see all the terms and collect like terms.

I need to check my answer.

Check: The left side is $3\left(-\frac{2}{11}\right) - \left(-\frac{2}{11} + 2\right) + 11\left(-\frac{2}{11}\right) = -\frac{6}{11} - \frac{20}{11} - 2 = -\frac{48}{11}$.

The right side is $\frac{1}{2}\left(4\left(-\frac{2}{11}\right) - 8\right) = \frac{1}{2}\left(-\frac{8}{11} - \frac{88}{11}\right) = \frac{1}{2}\left(\frac{96}{11}\right) = -\frac{48}{11}$. *Since* $-\frac{48}{11} = -\frac{48}{11}$, $x = -\frac{2}{11}$ *is the solution.*

2. $5(2 + x) - 4 = 81$

I need to use the distributive property to each term inside the parentheses only but not to the -4.

$$\begin{aligned}
5(2 + x) - 4 &= 81 \\
10 + 5x - 4 &= 81 \\
5x + 6 &= 81 \\
5x + 6 - 6 &= 81 - 6 \\
5x &= 75 \\
x &= 15
\end{aligned}$$

I can check this answer mentally.

3. $6x + \frac{1}{3}(9x + 5) = 10x + \frac{13}{3} - (x + 1)$

$$6x + \frac{1}{3}(9x + 5) = 10x + \frac{13}{3} - (x + 1)$$
$$6x + 3x + \frac{5}{3} = 10x + \frac{13}{3} - x - 1$$
$$9x + \frac{5}{3} = 9x + \frac{10}{3}$$
$$9x - 9x + \frac{5}{3} = 9x - 9x + \frac{10}{3}$$
$$\frac{5}{3} \neq \frac{10}{3}$$

This is an untrue sentence; therefore, this equation has no solution.

This equation has no solution.

G8-M4-Lesson 7: Classification of Solutions

1. Give a brief explanation as to what kind of solution(s) you expect for the linear equation $12x + 7 = -3(9 - 5x)$. Transform the equation into a simpler form if necessary.

The coefficients of x are different and so are the constants.

$$12x + 7 = -3(9 - 5x)$$
$$12x + 7 = -27 + 15x$$

This equation will have a unique solution.

After I use the distributive property on the right side, the coefficients of x are different ($12 \neq 15$), and the constants are different ($7 \neq -27$) on each side. This means the equation will have a unique solution.

2. Give a brief explanation as to what kind of solution(s) you expect for the linear equation $18\left(\frac{1}{2} + \frac{1}{3}x\right) = 6x + 9$. Transform the equation into a simpler form if necessary.

$$18\left(\frac{1}{2} + \frac{1}{3}x\right) = 6x + 9$$
$$9 + 6x = 6x + 9$$

This is an identity under the distributive property. Therefore, this equation will have infinitely many solutions.

After I use the distributive property on the left side, the coefficients of x are the same ($6 = 6$), and the constants are the same ($9 = 9$) on each side. This means the equation will have infinitely many solutions.

3. Give a brief explanation as to what kind of solution(s) you expect for the linear equation $5(2x + 4) = 2(5x - 10)$. Transform the equation into a simpler form if necessary.

$$5(2x + 4) = 2(5x - 10)$$
$$10x + 20 = 10x - 20$$

The coefficients of x are the same, but the constants are different. Therefore, this equation has no solutions.

After I use the distributive property on both sides of the equation, the coefficients of x are the same ($10 = 10$), and the constants are different ($20 \neq -20$) on each side. This means the equation will have no solution.

G8-M4-Lesson 8: Linear Equations in Disguise

Solve the following equations of rational expressions, if possible. If the equation cannot be solved, explain why.

1. $\frac{x+5}{-2} = \frac{3-x}{7}$

$$\begin{aligned} \frac{x+5}{-2} &= \frac{3-x}{7} \\ -2(3-x) &= (x+5)7 \\ -6+2x &= 7x+35 \\ -6+2x-2x &= 7x-2x+35 \\ -6 &= 5x+35 \\ -6-35 &= 5x+35-35 \\ -41 &= 5x \\ -\frac{41}{5} &= x \end{aligned}$$

I can multiply each numerator by the other fraction's denominator. I put the expressions with more than one term in parentheses so that I remember to use the distributive property.

2. $\frac{12}{x-3} = \frac{4}{x+2}$

$$\begin{aligned} \frac{12}{x-3} &= \frac{4}{x+2} \\ 12(x+2) &= (x-3)4 \\ 12x+24 &= 4x-12 \\ 12x-4x+24 &= 4x-4x-12 \\ 8x+24 &= -12 \\ 8x+24-24 &= -12-24 \\ 8x &= -36 \\ \frac{8}{8}x &= -\frac{36}{8} \\ x &= -\frac{9}{2} \end{aligned}$$

I used the distributive property, and now I see that I have a linear equation that I can solve using properties of equalities that I learned earlier in the module in Lesson 4.

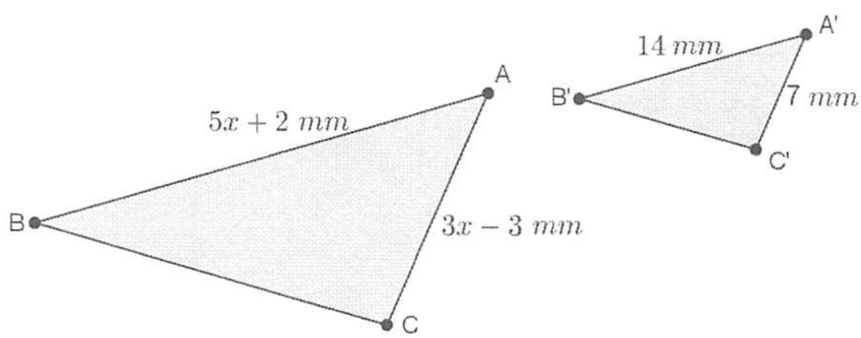
I can rewrite the fraction $-\frac{36}{8}$ as $-\frac{9}{2}$ because they are equivalent.

3. $\frac{\frac{1}{3}x - 2}{8} = \frac{4x}{9}$

$$\frac{\frac{1}{3}x - 2}{8} = \frac{4x}{9}$$
$$\left(\frac{1}{3}x - 2\right)9 = 8(4x)$$
$$3x - 18 = 32x$$
$$3x - 3x - 18 = 32x - 3x$$
$$-18 = 29x$$
$$-\frac{18}{29} = x$$

I could write the equation as $8(4x) = 9\left(\frac{1}{3}x - 2\right)$ because when I distribute, I will get $32x = 3x - 18$. When I use the properties of equalities, my answer will be the same, $x = -\frac{18}{29}$.

4. In the diagram below, $\triangle ABC \sim \triangle A'B'C'$. Determine the lengths of $\overline{AB}$ and $\overline{AC}$.



Since I know the triangles are similar, I can write a proportion using corresponding sides.

$$\frac{5x + 2}{14} = \frac{3x - 3}{7}$$
$$7(5x + 2) = 14(3x - 3)$$
$$35x + 14 = 42x - 42$$
$$35x - 35x + 14 = 42x - 35x - 42$$
$$14 = 7x - 42$$
$$14 + 42 = 7x - 42 + 42$$
$$56 = 7x$$
$$8 = x$$

I need to use my answer to determine the side lengths of the triangle.

The length of $\overline{AB}$ is $(5(8) + 2)$ mm $= 42$ mm, and the length of $\overline{AC}$ is $(3(8) - 3)$ mm $= 21$ mm.

G8-M4-Lesson 9: An Application of Linear Equations

1. You forward a blog that you found online to five of your friends. They liked it so much that they forwarded it on to two of their friends, who then forwarded it on to two of their friends, and so on. The number of people who saw the blog is shown below. Let S_1 represent the number of people who saw the blog after one step, let S_2 represent the number of people who saw the blog after two steps, and so on.

$$S_1 = 5$$
$$S_2 = 5 + 5 \cdot 2$$
$$S_3 = 5 + 5 \cdot 2 + 5 \cdot 2^2$$
$$S_4 = 5 + 5 \cdot 2 + 5 \cdot 2^2 + 5 \cdot 2^3$$

I will start with S_2 since $S_1 = 5$ and try to manipulate S_2 into an equation that contains S_2.

a. Find the pattern in the equations.

By adding $5 \cdot 2^2$, I can use the distributive property to get a linear equation in S_2.

$$S_2 = 5 + 5 \cdot 2$$
$$S_2 - 5 = 5 \cdot 2$$
$$S_2 - 5 + 5 \cdot 2^2 = 5 \cdot 2 + 5 \cdot 2^2$$
$$S_2 - 5 + 5 \cdot 2^2 = 2(5 + 5 \cdot 2)$$
$$S_2 - 5 + 5 \cdot 2^2 = 2S_2$$

$$S_3 = 5 + 5 \cdot 2 + 5 \cdot 2^2$$
$$S_3 - 5 = 5 \cdot 2 + 5 \cdot 2^2$$
$$S_3 - 5 + 5 \cdot 2^3 = 5 \cdot 2 + 5 \cdot 2^2 + 5 \cdot 2^3$$
$$S_3 - 5 + 5 \cdot 2^3 = 2(5 + 5 \cdot 2 + 5 \cdot 2^2)$$
$$S_3 - 5 + 5 \cdot 2^3 = 2S_3$$

By adding $5 \cdot 2$ raised to the power of the step number, I can use the distributive property to get a linear equation in terms of that step number.

I don't want to multiply out any of the terms so that I can see the pattern better.

$$S_4 = 5 + 5 \cdot 2 + 5 \cdot 2^2 + 5 \cdot 2^3$$
$$S_4 - 5 = 5 \cdot 2 + 5 \cdot 2^2 + 5 \cdot 2^3$$
$$S_4 - 5 + 5 \cdot 2^4 = 5 \cdot 2 + 5 \cdot 2^2 + 5 \cdot 2^3 + 5 \cdot 2^4$$
$$S_4 - 5 + 5 \cdot 2^4 = 2(5 + 5 \cdot 2 + 5 \cdot 2^2 + 5 \cdot 2^3)$$
$$S_4 - 5 + 5 \cdot 2^4 = 2S_4$$

b. Assuming the trend continues, how many people will have seen the blog after 8 steps?

I want to use the properties of equality to get S_8 on one side and the constants on the other side of the equal sign and use the distributive property.

$$S_8 - 5 + 5 \cdot 2^8 = 2S_8$$
$$S_8 - 2S_8 = 5 - 5 \cdot 2^8$$
$$S_8(1-2) = 5 - 5 \cdot 2^8$$
$$S_8(1-2) = 5(1-2^8)$$
$$S_8 = \frac{5(1-2^8)}{(1-2)}$$

I multiplied out on the last step.

$$S_8 = 1{,}275$$

After 8 steps, 1,275 people will have seen the blog.

c. How many people will have seen the blog after n steps?

$$S_n = \frac{5(1-2^n)}{(1-2)}$$

I see a pattern from the work I have done.

2. The length of a rectangle is 4 more than 2 times the width. If the perimeter of the rectangle is 20.6 cm, what is the area of the rectangle?

Let x represent the width of the rectangle. Then the length of the rectangle is $4 + 2x$.

Since I know the perimeter, I will write my equation in terms of perimeter. Perimeter of a rectangle means I need to add twice the width to twice the length, $P = 2w + 2l$.

$$2(4 + 2x) + 2x = 20.6$$
$$8 + 4x + 2x = 20.6$$
$$8 + 6x = 20.6$$
$$6x = 12.6$$
$$x = \frac{12.6}{6}$$
$$x = 2.1$$

The problem asked for the area of the rectangle. Area of a rectangle means I have to multiply the length and width.

The width of the rectangle is 2.1 cm, and the length is $(4 + 2(2.1))$ cm $= 8.2$ cm, so the area is 17.22 cm^2.

3. Each month, Gilbert pays \$42 to his phone company just to use the phone. Each text he sends costs him an additional \$0.15. In June, his phone bill was \$162.75. In July, his phone bill was \$155.85. How many texts did he send each month?

Let x be the number of texts he sent in June.

$$42 + 0.15x = 162.75$$
$$0.15x = 120.75$$
$$x = \frac{120.75}{0.15}$$
$$x = 805$$

He sent 805 texts in June.

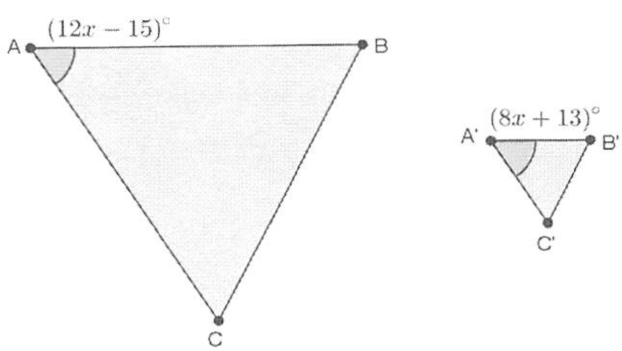
Let y be the number of texts he sent in July.

$$42 + 0.15y = 155.85$$
$$0.15y = 113.85$$
$$y = \frac{113.85}{0.15}$$
$$y = 759$$

He sent 759 texts in July.

I am using a different letter to define my variable because the number of texts for July is different than the number of texts for June since the cost was different for both months.

4. In the diagram below, $\triangle ABC \sim \triangle A'B'C'$. Determine the measure of $\angle A$.



Since the triangles are similar, the angles are equal in measure.

$$12x - 15 = 8x + 13$$
$$12x - 8x - 15 = 8x - 8x + 13$$
$$4x - 15 = 13$$
$$4x - 15 + 15 = 13 + 15$$
$$4x = 28$$
$$x = 7$$

The measure of $\angle A$ is $(12(7) - 15)^\circ = 69^\circ$.

G8-M4-Lesson 10: A Critical Look at Proportional Relationships

1. Jurgen types a paper for his Humanities class at a constant speed. He types 12 pages, and it took him 66 minutes.

   a. What fraction represents his constant speed, C?

   $$C = \frac{12}{66} = \frac{2}{11}$$

   To write the fraction for his constant speed, I have to compare the number of pages typed to the interval of time spent typing.

   b. Write the fraction that represents his constant speed, C, if he types y pages in 24 minutes.

   $$C = \frac{y}{24}$$

   c. Write a proportion using the fractions from parts (a) and (b) to determine how many pages he types after 24 minutes. Round your answer to the hundredths place.

   $$\frac{2}{11} = \frac{y}{24}$$
   $$2(24) = 11(y)$$
   $$48 = 11(y)$$
   $$\frac{1}{11}(48) = \frac{1}{11}(11)y$$
   $$4.36 \approx y$$

   Jurgen types approximately 4.36 pages in 24 minutes.

   d. Write a two-variable equation to represent how many pages Jurgen can type over any time interval.

   Let y represent the number of pages typed. Let x represent the number of minutes typed.

   $$\frac{2}{11} = \frac{y}{x}$$
   $$2(x) = 11(y)$$
   $$\frac{1}{11}(2)x = \frac{1}{11}(11)y$$
   $$\frac{2}{11}x = y$$

   When I write a two-variable equation, I have to remember to define my variables.

2. Parker runs at a constant speed of 6.25 miles per hour.

   a. If he runs for y miles and it takes him x hours, write the two-variable equation to represent the number of miles Parker can run in x hours.

   Let y represent the number of miles run. Let x represent the number of hours run.

$$\frac{6.25}{1} = \frac{y}{x}$$
$$6.25x = y$$

   b. Parker has been training for a marathon by running to the school 11 miles from his house, then to the park 2 miles from the school, and then returning home, which is 14 miles from the park. Assuming he runs at a constant speed the entire time, how long will it take him to get back home after running his route? Round your answer to the hundredths place.

   Total miles: $11 + 2 + 14 = 27$. ***Let x be the number of hours run.***

$$6.25x = 27$$
$$\frac{1}{6.25}(6.25)x = \frac{1}{6.25}(27)$$
$$x = 4.32$$

   It will take Parker 4.32 ***hours to run*** 27 ***miles.***

3. Jared walks from baseball practice to his aunt's house, a distance of 6 miles, in 90 minutes. Assuming he walks at a constant speed, C, how far does he walk in 20 minutes? Round your answer to the hundredths place.

   Let y represent the number of miles walked.

   Since $\frac{6}{90} = C$ ***and*** $\frac{y}{20} = C$, ***then***

$$\frac{6}{90} = \frac{y}{20}$$
$$6(20) = 90y$$
$$120 = 90y$$
$$\frac{1}{90}(120) = \frac{1}{90}(90)y$$
$$\frac{120}{90} = y$$
$$1.33 \approx y$$

   Jared walks approximately 1.33 ***miles in*** 20 ***minutes.***

4. Sammy bikes 3 miles every night for exercise. It takes him exactly 1.75 hours to finish his ride.

   a. Assuming he rides at a constant rate, write an equation that represents how many miles, y, Sammy can ride in x hours.

$$\frac{3}{1.75} = \frac{y}{x}$$
$$3x = 1.75y$$
$$\frac{1}{1.75}(3)x = \frac{1}{1.75}(1.75)y$$
$$\frac{3}{1.75}x = y$$

I don't need to define my variables for this problem because they have already done it in the problem.

   b. Use your equation from part (a) to complete the table below. Use a calculator, and round all values to the hundredths place.

x (hours)	Linear Equation in y: $\frac{3}{1.75}x = y$	y (miles)
0.25	$\frac{3}{1.75}(0.25) = y$	0.43
0.5	$\frac{3}{1.75}(0.5) = y$	0.86
0.75	$\frac{3}{1.75}(0.75) = y$	1.29
1	$\frac{3}{1.75}(1) = y$	1.71
3	$\frac{3}{1.75}(3) = y$	5.14

G8-M4-Lesson 11: Constant Rate

1. A bus travels at a constant rate of 40 miles per hour.

   What is the distance, d, in miles, that the bus travels in t hours?

   Let C be the constant rate the bus travels. Then, $\frac{40}{1} = C$, and $\frac{d}{t} = C$; therefore, $\frac{40}{1} = \frac{d}{t}$.

   $$\frac{40}{1} = \frac{d}{t}$$
   $$d = 40t$$

   If I can write two fractions, each equal to the constant rate, C, then I can use the proportional relationship to solve for d.

2. A teenage boy named Harry can consume 8 hot dogs in 1.25 hours. Assume that the young man eats at a constant rate.

   a. How many hot dogs, y, can be consumed by Harry in t hours?

   Let C be the constant rate Harry eats hot dogs. Then, $\frac{8}{1.25} = C$, and $\frac{y}{t} = C$; therefore, $\frac{8}{1.25} = \frac{y}{t}$.

   $$\frac{8}{1.25} = \frac{y}{t}$$
   $$1.25y = 8t$$
   $$\frac{1.25}{1.25}y = \frac{8}{1.25}t$$
   $$y = 6.4t$$

   b. Pretend that he can eat every hour of every day for a week. How many hot dogs would Harry consume?

   24 hours a day for 7 days is a total of 168 hours.

   $$y = 6.4t$$
   $$y = 6.4(168)$$
   $$y = 1{,}075.2$$

   Once I figure out how many hours are in a week, I can use my equation from part (a) to determine the answer.

   Harry would consume about $1{,}075$ hot dogs in one week.

3. Your cell phone company charges at a constant rate. The company charges $1.00 for 4 minutes of use.

   a. Write an equation to represent the number of dollars, d, that will be charged over any time interval, t.

   Let C be the constant rate charged per minute. Then, $\frac{1.00}{4} = C$, and $\frac{d}{t} = C$; therefore, $\frac{1.00}{4} = \frac{d}{t}$.

$$\frac{1}{4} = \frac{d}{t}$$

$$4d = 1t$$

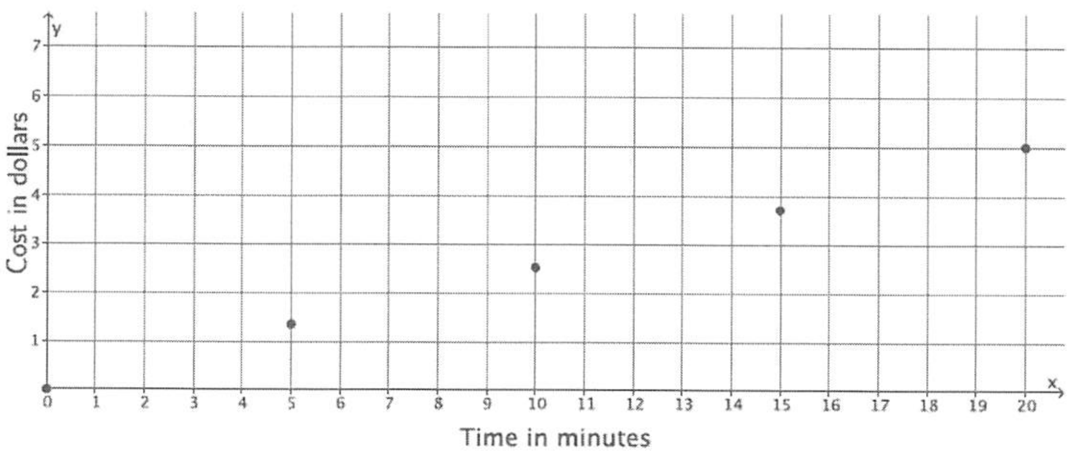
$$\frac{1}{4}(4)d = \frac{1}{4}(1)t$$

$$d = 0.25t$$

   b. Complete the table below.

t (time in minutes)	Linear Equation: $d = 0.25t$	d (cost in dollars)
0	$d = 0.25(0)$	0
5	$d = 0.25(5)$	1.25
10	$d = 0.25(10)$	2.50
15	$d = 0.25(15)$	3.75
20	$d = 0.25(20)$	5.00

   c. Graph the data as points on a coordinate plane.



d. You used your phone for 18 minutes. About how much will your bill be? Explain.

It will cost between $\$3.75$ *and* $\$5$. *I located* 18 *on the* x*-axis because that is the number of minutes I used. That* x*-value is between the known costs for* 15 *minutes and* 20 *minutes. So my bill will probably be closer to* $\$5$ *because* 18 *is closer to* 20 *than to* 15.

G8-M4-Lesson 12: Linear Equations in Two Variables

1. Consider the linear equation $x - \frac{2}{5}y = 4$.

   a. Will you choose to fix values for x or y? Explain.

   If I fix values for y, it will make the computations easier. Solving for x can be done in one step.

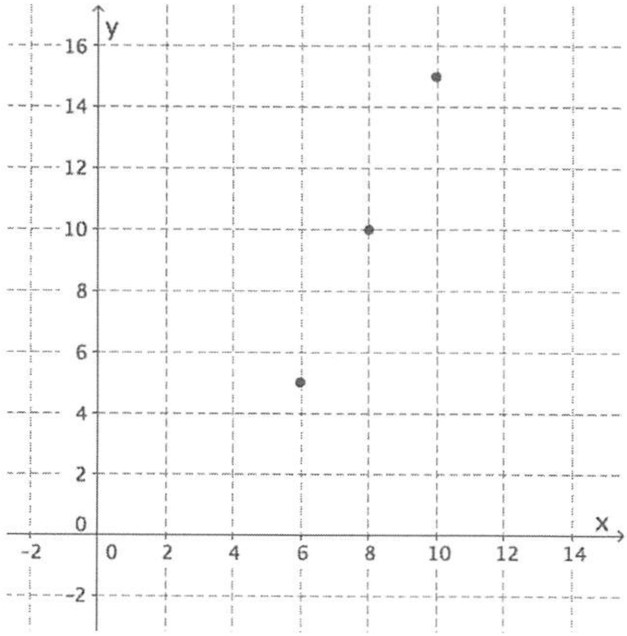
   b. Are there specific numbers that would make your computational work easier? Explain.

   Values for y that are multiples of 5 will make the computations easier. When I multiply $\frac{2}{5}$ by a multiple of 5, I will get a whole number.

   c. Find three solutions to the linear equation $x - \frac{2}{5}y = 4$, and plot the solutions as points on a coordinate plane.

I'll use the numbers 5, 10, and 15 for y in my table. Once substituted into the equation, I'll get the values for x. Then each pair of x and y will be a point on my graph.

x	Linear Equation: $x - \frac{2}{5}y = 4$	y
6	$x - \frac{2}{5}(5) = 4$ $x - 2 = 4$ $x = 6$	5
8	$x - \frac{2}{5}(10) = 4$ $x - 4 = 4$ $x = 8$	10
10	$x - \frac{2}{5}(15) = 4$ $x - 6 = 4$ $x = 10$	15



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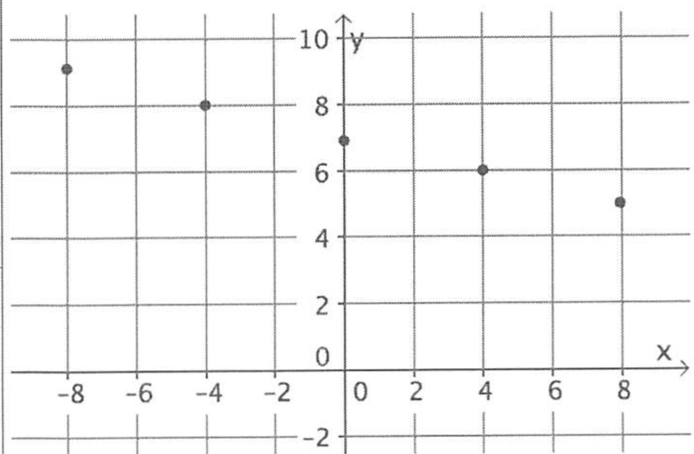
G8-M4-Lesson 13: The Graph of a Linear Equation in Two Variables

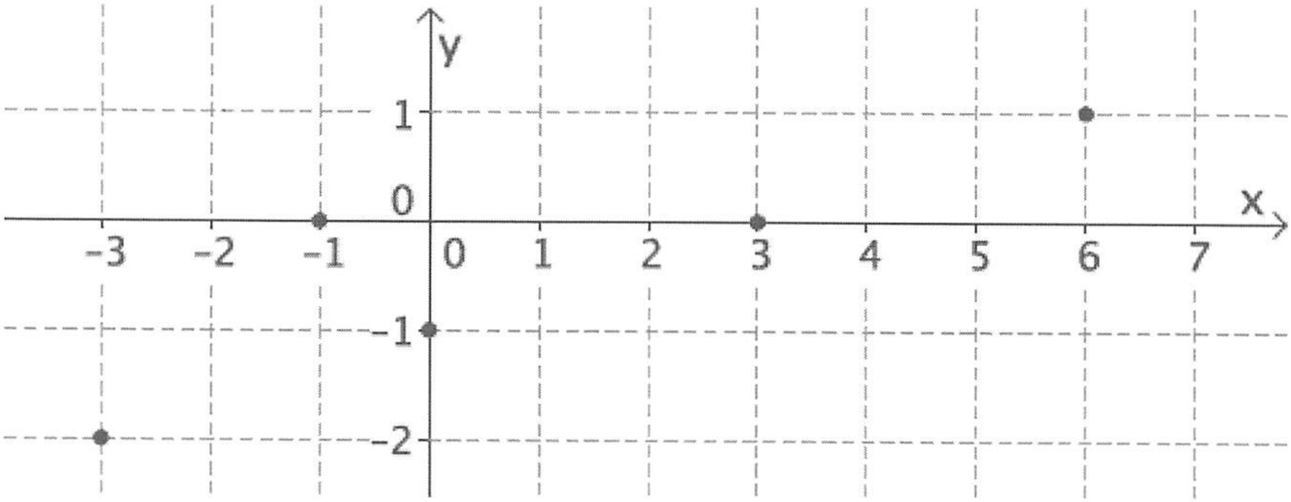
1. Find at least five solutions to the linear equation $\frac{1}{4}x + y = 7$, and plot the points on a coordinate plane. What shape is the graph of the linear equation taking?

I should choose values for x that are multiples of 4. I should also be sure to select some positive values for x as well as negative.

x	$\frac{1}{4}x + y = 7$	y
-8	$\frac{1}{4}(-8) + y = 7$ $-2 + y = 7$ $-2 + 2 + y = 7 + 2$ $y = 9$	9
-4	$\frac{1}{4}(-4) + y = 7$ $-1 + y = 7$ $-1 + 1 + y = 7 + 1$ $y = 8$	8
0	$\frac{1}{4}(0) + y = 7$ $0 + y = 7$ $y = 7$	7
4	$\frac{1}{4}(4) + y = 7$ $1 + y = 7$ $1 - 1 + y = 7 - 1$ $y = 6$	6
8	$\frac{1}{4}(8) + y = 7$ $2 + y = 7$ $2 - 2 + y = 7 - 2$ $y = 5$	5

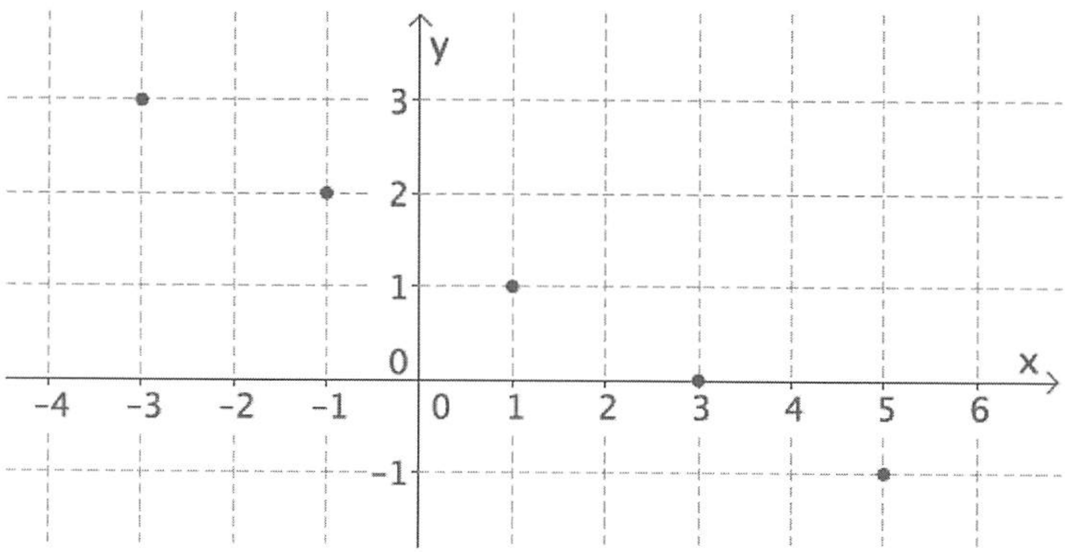


2. Can the following points be on the graph of the equation $x - 3y = 3$? Explain



The graph shown contains the point $(-1, 0)$*. If* $(-1, 0)$ *is on the graph of the linear equation, then it will be a solution to the equation. It is not; therefore, the point cannot be on the graph of the equation, which means the graph shown cannot be the graph of the equation* $x - 3y = 3$*.*

3. Can the following points be on the graph of the equation $2x + 4y = 6$? Explain



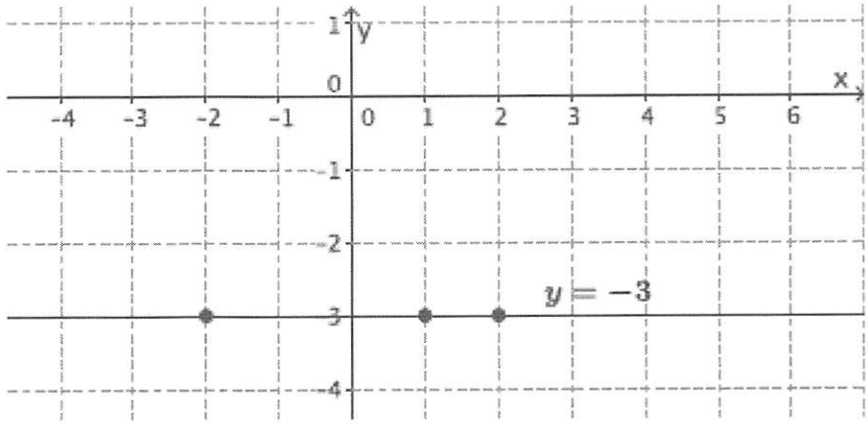
Yes, this graph is of the equation $2x + 4y = 6$ *because each point on the graph represents a solution to the linear equation* $2x + 4y = 6$*.*

G8-M4-Lesson 14: Graph of a Linear Equation—Horizontal and Vertical Lines

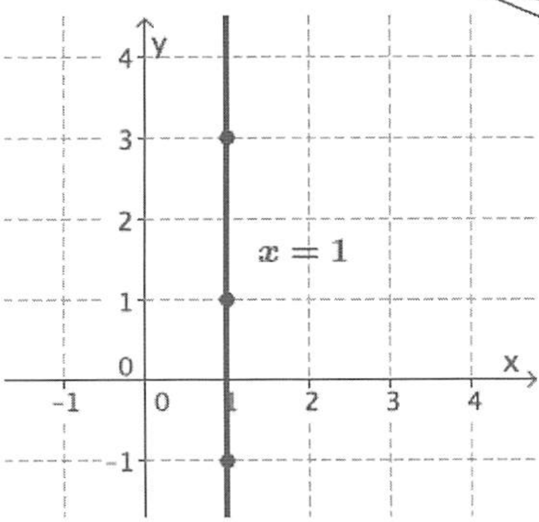
1. Graph the two-variable linear equation $ax + by = c$, where $a = 0$, $b = -2$, and $c = 6$.

$$ax + by = c$$
$$0x + (-2)y = 6$$
$$-2y = 6$$
$$y = -3$$

I'm not sure how to graph this, so I'll find some solutions using a table like in the last lesson.



2. Graph the linear equation $x = 1$.



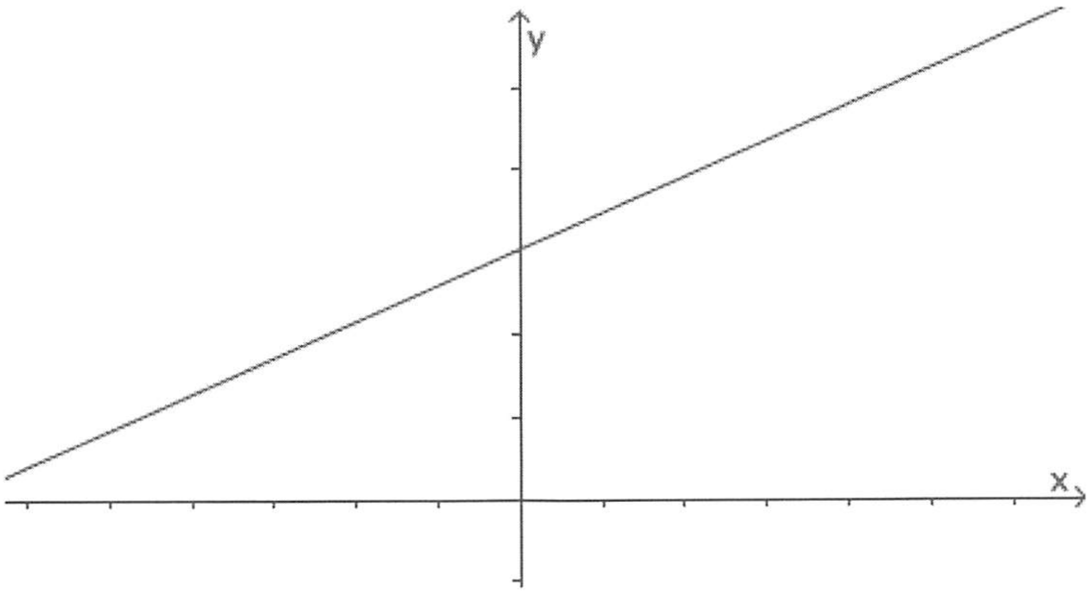
I know that this will either be a horizontal or vertical line. Since the equation is $x = 1$, that means that no matter what value I choose for y, the x-value will always be one.

3. Explain why the graph of a linear equation in the form of $x = c$ is the vertical line, parallel to the y-axis passing through the point $(c, 0)$.

   The graph of $x = c$ passes through the point $(c\ , 0)$, which means the graph of $x = c$ cannot be parallel to the x-axis because the graph intersects it. For that reason, the graph of $x = c$ must be a vertical line parallel to the y-axis.

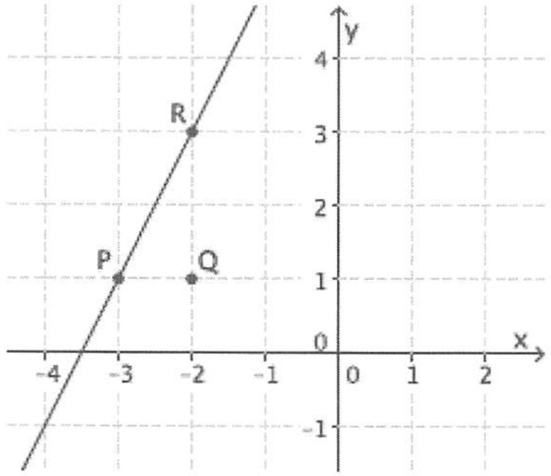
G8-M4-Lesson 15: The Slope of a Non-Vertical Line

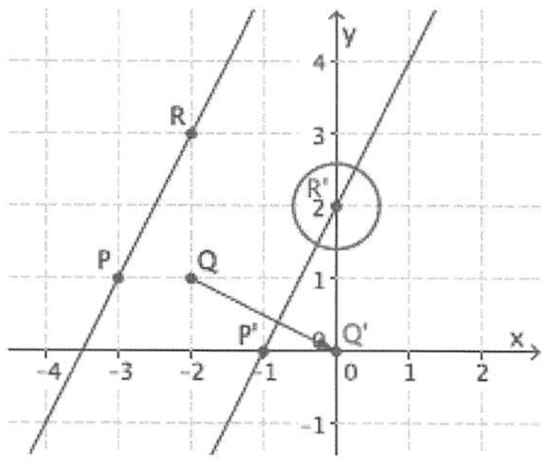
1. Does the graph of the line shown below have a positive or negative slope? Explain



The graph of this line has a positive slope. It is left-to-right inclining, which is an indication of positive slope.

2. What is the slope of this non-vertical line? Use your transparency if needed.





The slope of this line is 2*, so* $m = 2$*.*

Since the distance between points P and Q is 1 unit, I can trace everything onto a transparency and map point Q to the origin. The location of the translated point R gives me the slope of the line.

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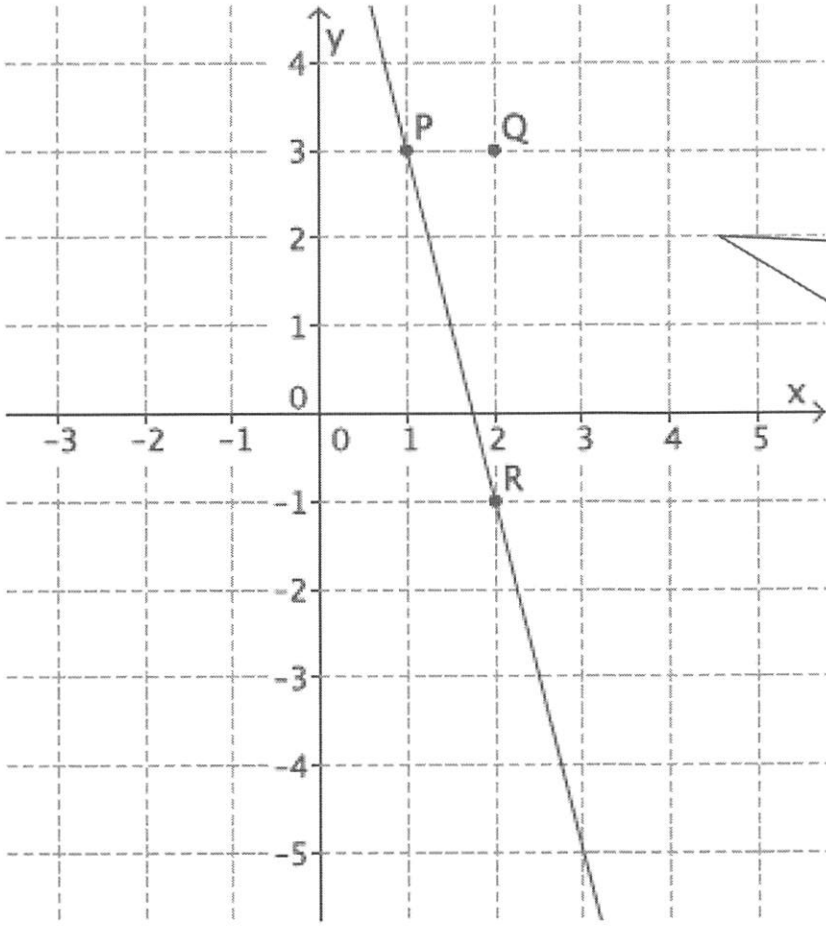
3. What is the slope of this non-vertical line? Use your transparency if needed.

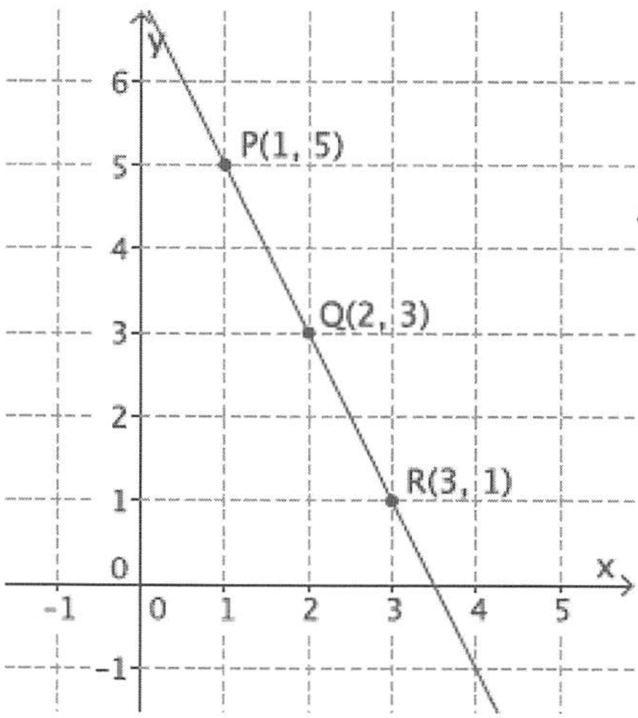


I can tell by the line that the slope will be negative. Just like I did in the last problem, I will use my transparency and translation to figure out the number that represents the slope.

The slope of this line is -4, so $m = -4$.

G8-M4-Lesson 16: The Computation of the Slope of a Non-Vertical Line

1. Calculate the slope of the line using two different pairs of points.



I need to choose three points on the line. The points $P(p_1, p_2)$ and $Q(q_1, q_2)$ are used in my first slope equation. I need to remember that no matter how the slope is written, the difference in the 2^{nd} values (y-values) is in the numerator of the slope, and the difference in the 1^{st} values (x-values) is in the denominator of the slope. The equation $m = \frac{q_2 - p_2}{p_1 - q_1}$ would be wrong since the values of point Q do not come first in each difference.

$$
\begin{aligned}
m &= \frac{p_2 - q_2}{p_1 - q_1} \\
&= \frac{5 - 3}{1 - 2} \\
&= \frac{2}{-1} \\
&= -2
\end{aligned}
$$

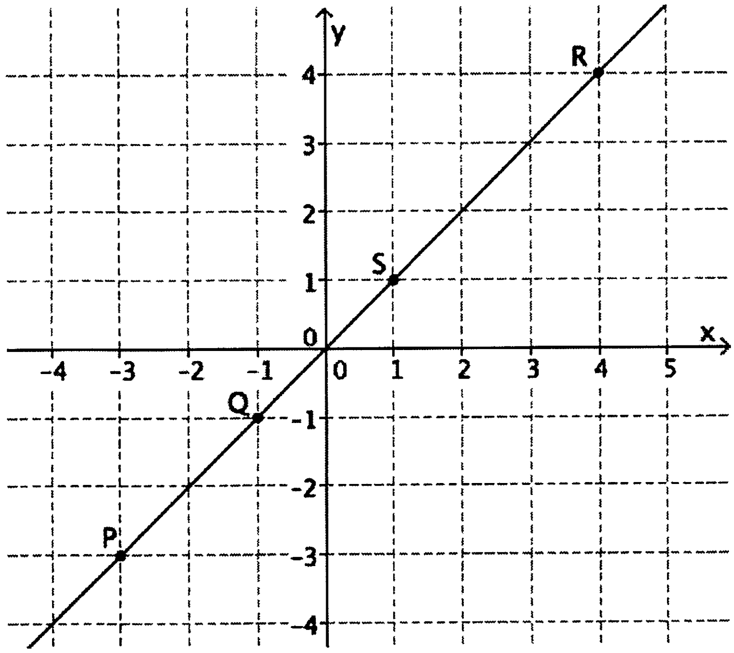
$$
\begin{aligned}
m &= \frac{r_2 - q_2}{r_1 - q_1} \\
&= \frac{1 - 3}{3 - 2} \\
&= \frac{-2}{1} \\
&= -2
\end{aligned}
$$

2. Calculate the slope of the line using two different pairs of points.

   a. Select any two points on the line to compute the slope.

   Let the two points be $P(-3,-3)$ *and* $Q(-1,-1)$.

$$m = \frac{p_2 - q_2}{p_1 - q_1} = \frac{-3-(-1)}{-3-(-1)} = \frac{-2}{-2} = 1$$



   b. Select two different points on the line to calculate the slope.

   Let the two points be $S(1,1)$ *and* $R(4,4)$.

$$m = \frac{s_2 - r_2}{s_1 - r_1} = \frac{1-4}{1-4} = \frac{-3}{-3} = 1$$

   c. What do you notice about your answers in parts (a) and (b)? Explain.

   The slopes are equal in parts (a) and (b). This is true because of what we know about similar triangles. The slope triangle that is drawn between the two points selected in part (a) is similar to the slope triangle that is drawn between the two points in part (b) by the AA criterion. Then, because the corresponding sides of similar triangles are equal in ratio, the slopes are equal.

3. Your teacher tells you that a line goes through the points $\left(1, \frac{3}{4}\right)$ and $(-2, -3)$.

   a. Calculate the slope of this line.

$$m = \frac{p_2 - r_2}{p_1 - r_1}$$
$$= \frac{\frac{3}{4} - (-3)}{1 - (-2)}$$
$$= \frac{3\frac{3}{4}}{3}$$
$$= \frac{5}{4}$$

   b. Do you think the slope will be the same if the order of the points is reversed? Verify by calculating the slope, and explain your result.

   The slope should be the same because we are joining the same two points. Since the slope of a line can be computed using any two points on the same line, it makes sense that it does not matter which point we name as P and which point we name as R.

$$m = \frac{r_2 - p_2}{r_1 - p_1}$$
$$= \frac{-3 - \frac{3}{4}}{-2 - 1}$$
$$= \frac{-3\frac{3}{4}}{-3}$$
$$= \frac{5}{4}$$

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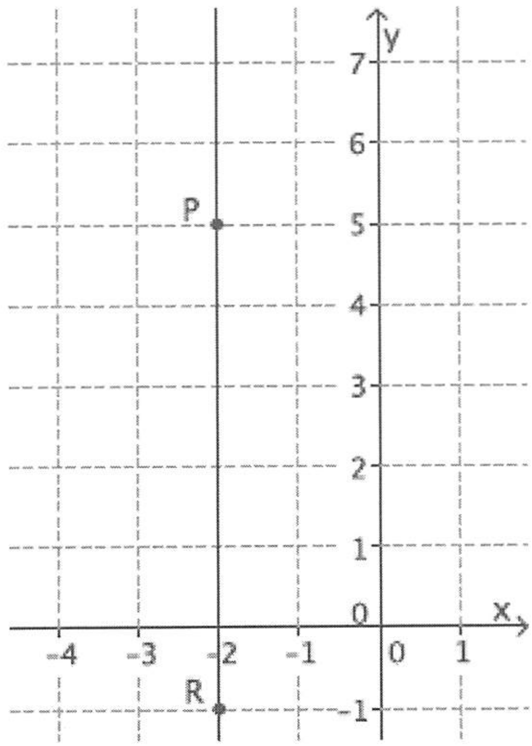
4. Each of the lines in the lesson was non-vertical. Consider the slope of a vertical line, $x = -2$. Select two points on the line to calculate slope. Based on your answer, why do you think the topic of slope focuses only on non-vertical lines?

$$m = \frac{r_2 - p_2}{r_1 - p_1}$$
$$= \frac{-1 - 5}{-2 - (-2)}$$
$$= \frac{-6}{0}$$



The computation of slope using the formula leads to a fraction with zero as its denominator, which is undefined. The topic of slope does not focus on vertical lines because the slope of a vertical line is undefined.

G8-M4-Lesson 17: The Line Joining Two Distinct Points of the Graph $y = mx+b$ Has Slope m

1. Solve the following equation for y: $-3x + 9y = 18$. Then, answer the questions that follow.

$$-3x + 9y = 18$$
$$-3x + 3x + 9y = 18 + 3x$$
$$9y = 18 + 3x$$
$$\frac{9}{9}y = \frac{18}{9} + \frac{3}{9}x$$
$$y = 2 + \frac{1}{3}x$$
$$y = \frac{1}{3}x + 2$$

a. Based on your transformed equation, what is the slope of the linear equation $-3x + 9y = 18$?

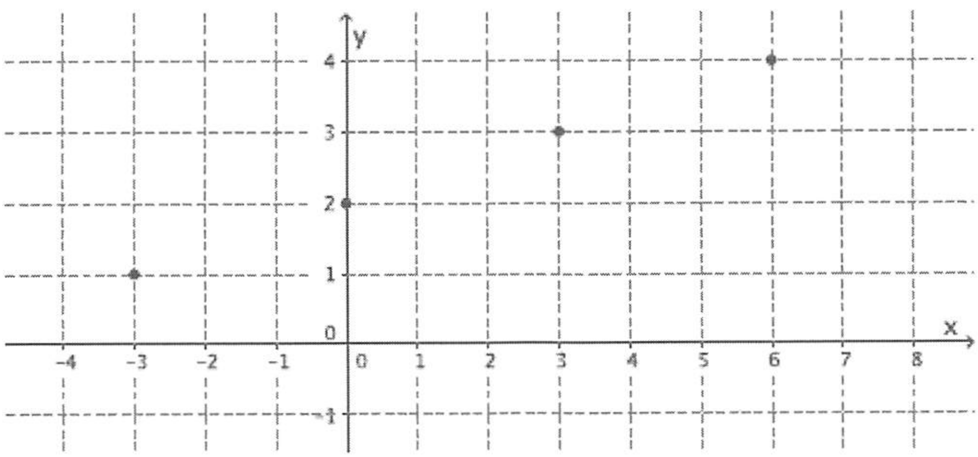
The slope is $\frac{1}{3}$.

b. Complete the table to find solutions to the linear equation.

x	Transformed Equation: $y = \frac{1}{3}x + 2$	y
-3	$y = \frac{1}{3}(-3) + 2$ $= -1 + 2$ $= 1$	1
0	$y = \frac{1}{3}(0) + 2$ $= 2$	2
3	$y = \frac{1}{3}(3) + 2$ $= 1 + 2$ $= 3$	3
6	$y = \frac{1}{3}(6) + 2$ $= 2 + 2$ $= 4$	4

Since the slope is a fraction, $\frac{1}{3}$, I need to choose x-values that are multiples of 3.

c. Graph the points on the coordinate plane.

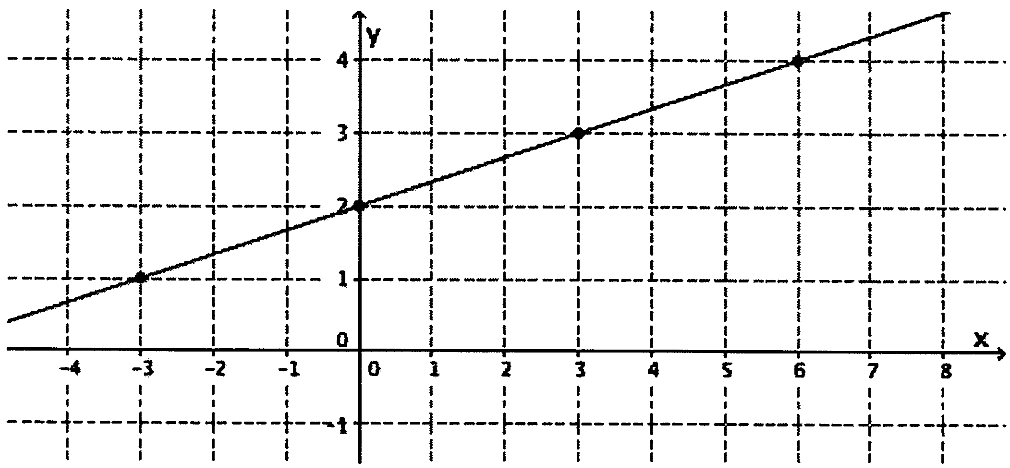


d. Find the slope between any two points.

Using the points $(-3, 1)$ *and* $(3, 3)$,

$$m = \frac{1-3}{-3-3}$$
$$= \frac{-2}{-6}$$
$$= \frac{1}{3}$$

e. The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of an equation of the form $y = mx + b$ that has slope m.



f. Note the location (ordered pair) that describes where the line intersects the y-axis.

$(0, 2)$ is the location where the line intersects the y-axis.

G8-M4-Lesson 18: There Is Only One Line Passing Through a Given Point with a Given Slope

Graph each equation on a separate pair of x- and y-axes. Students need graph paper to complete the Problem Set.

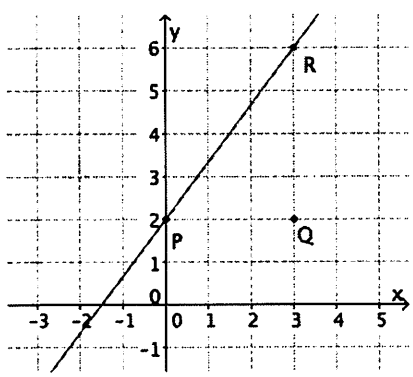
1. Graph the equation $y = \frac{4}{3}x + 2$.

I know the equation is in slope-intercept form, $y = mx + b$, the number m represents the slope of the graph, and the point $(0, b)$ is the location where the graph of the line intersects the y-axis.

   a. Name the slope and the y-intercept point.

   The slope is $m = \frac{4}{3}$, and the y-intercept point is $(0, 2)$.

   b. Graph the known point, and then use the slope to find a second point before drawing the line.



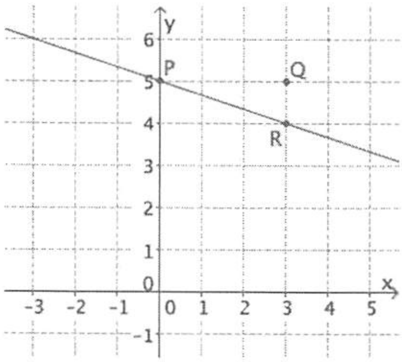
I know that $m = \frac{|QR|}{|PQ|}$. Since $|PQ| = 3$, I need to go 3 units from the right of point P, the y-intercept, to find point Q. Since $|QR| = 4$ and the slope is positive, I need to go <u>up</u> 4 units from point Q to find point R.

2. Graph the equation $y = -\frac{1}{3}x + 5$.

   a. Name the slope and the y-intercept point.

   The slope is $m = -\frac{1}{3}$, and the y-intercept point is $(0, 5)$.

b. Graph the known point, and then use the slope to find a second point before drawing the line.



I need to go three units to the right of point P and mark point Q. Since the slope is negative, I need to go <u>down</u> 1 unit from point Q to find point R.

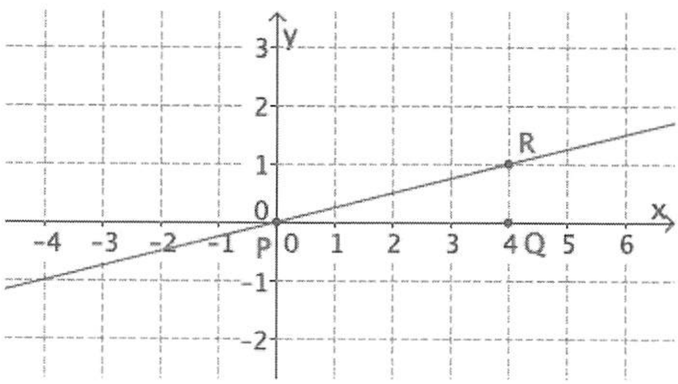
3. Graph the equation $y = \frac{1}{4}x$.

Rewriting the equation in slope-intercept form, $y = \frac{1}{4}x + 0$, helps me to see the y-intercept point is $(0,0)$.

a. Name the slope and the y-intercept point.

The slope is $m = \frac{1}{4}$, and the y-intercept point is $(0, 0)$.

b. Graph the known point, and then use the slope to find a second point before drawing the line.



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4. Graph the equation $2x + 2y = 2$.

   a. Name the slope and the y-intercept point.

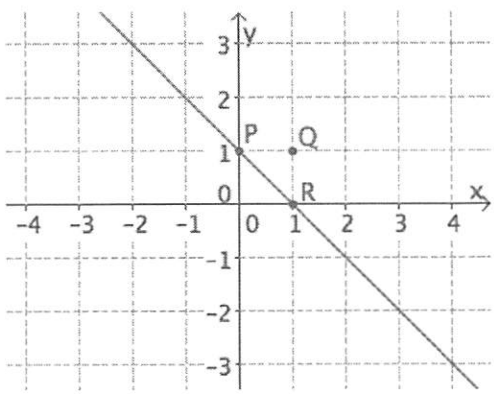
I need to rewrite the equation in slope-intercept form to help me name the slope and y-intercept point more easily.

$$2x + 2y = 2$$
$$2x - 2x + 2y = -2x + 2$$
$$2y = -2x + 2$$
$$\frac{2}{2}y = -\frac{2}{2}x + \frac{2}{2}$$
$$y = -x + 1$$

The slope -1 is equivalent to the fraction $-\frac{1}{1}$.

The slope is $m = -1$, and the y-intercept point is $(0, 1)$.

   b. Graph the known point, and then use the slope to find a second point before drawing the line.

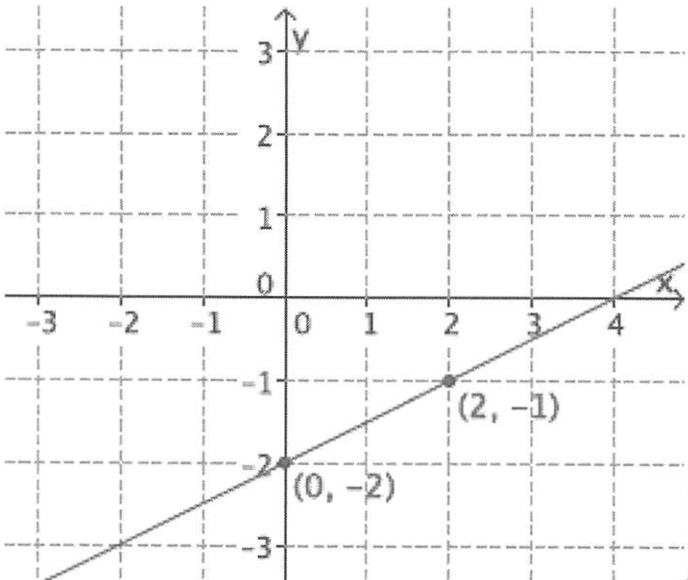


G8-M4-Lesson 19: The Graph of a Linear Equation in Two Variables Is a Line

Students need graph paper to complete the Problem Set.

1. Graph the equation: $y = \frac{1}{2}x - 2$.

This is a linear equation in slope-intercept form. I will use the slope $m = \frac{1}{2}$ and the y-intercept point $(0, -2)$ to graph the linear equation.



2. Graph the equation: $4x + 8y = 16$.

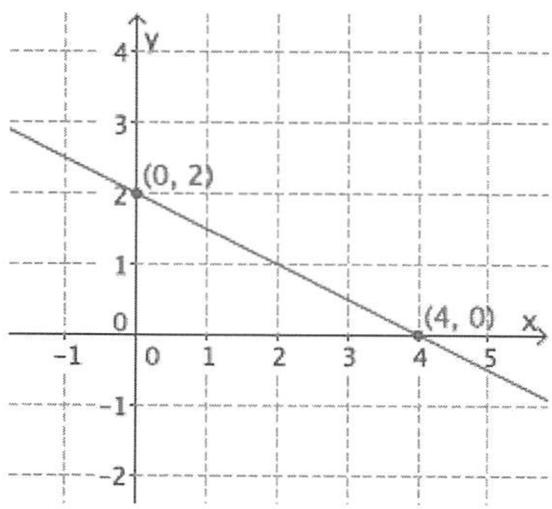
This is a linear equation in standard form. I will find the y-intercept point by replacing the x with 0. I will find the x-intercept point by replacing the y with 0.

$$4(0) + 8y = 16$$
$$8y = 16$$
$$y = 2$$

The y-intercept point is $(0, 2)$.

$$4x + 8(0) = 16$$
$$4x = 16$$
$$x = 4$$

The x-intercept point is $(4, 0)$.



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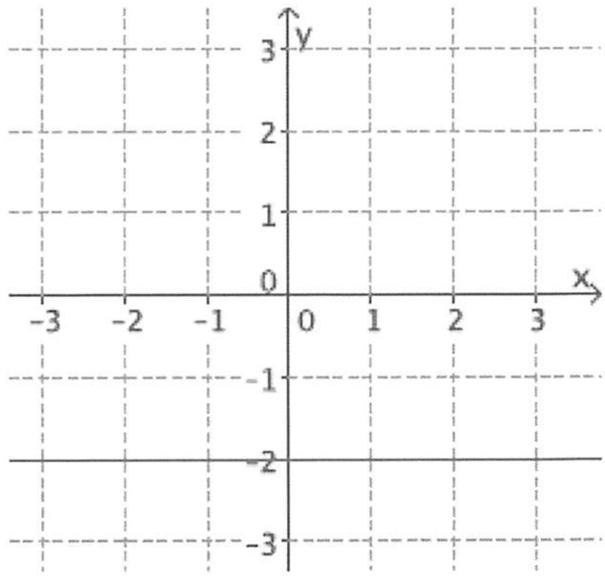
3. Graph the equation: $y = -2$. What is the slope of the graph of this line?



I remember that equations of the form $y = b$ are horizontal lines passing through the point $(0, b)$ where b is a constant.

The slope of this line is zero.

I can calculate the slope by using any two points on the graph of the line.

4. Is the graph of $x^2 - 6y = 11$ a line? Explain.

The graph of the given equation is not a line. The equation $6x^2 - 6y = 11$ is not a linear equation because the expression on the left side of the equal sign is not a linear expression. If this were a linear equation, then I would be sure that it graphs as a line, but because it is not, I am not sure what the graph of this equation would look like.

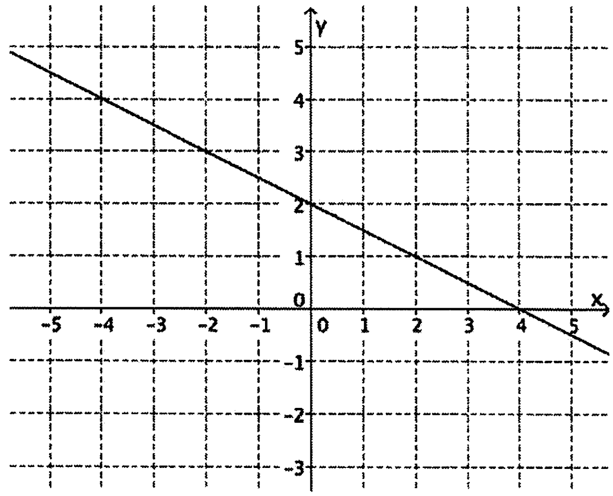
Linear expressions are constants like -1 or 5. Linear expressions can be a product of constants and an x like $5x$ or $-2x$, or a product of constants and a y like $9y$ or $-11y$.

G8-M4-Lesson 20: Every Line Is a Graph of a Linear Equation

1. Write the equation that represents the line shown.

$$y = -\frac{1}{2}x + 2$$

I identified point P as the y-intercept, which is $(0,2)$. I can use any point on the graph for point R, so I will use $(-4,4)$. Point Q will be $(-4,2)$. This will help me find the slope of $-\frac{2}{4}$, which is equivalent to $-\frac{1}{2}$. I will substitute the information into the slope-intercept form of the equation.



a. Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a, b, and c are integers, and a is not negative.

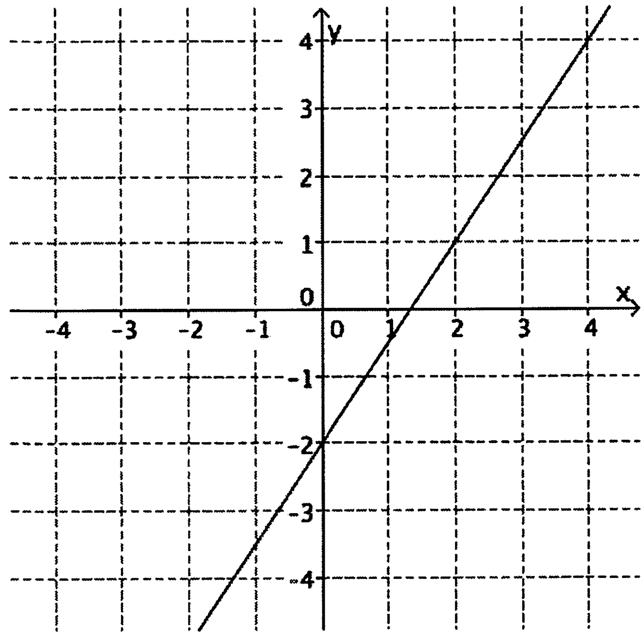
$$y = -\frac{1}{2}x + 2$$
$$\left(y = -\frac{1}{2}x + 2\right)2$$
$$2y = -x + 4$$
$$x + 2y = -x + x + 4$$
$$x + 2y = 4$$

What number can I multiply the equation by so that $-\frac{1}{2}$ will become an integer?

2. Write the equation that represents the line shown.

$$y = \frac{3}{2}x - 2$$

I need to calculate the slope and determine the y-intercept like I did in Problem 1.



a. Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a, b, and c are integers, and a is not negative.

$$y = \frac{3}{2}x - 2$$

$$\left(y = \frac{3}{2}x - 2\right)2$$

$$2y = 3x - 4$$

$$-3x + 2y = 3x - 3x - 4$$

$$-3x + 2y = -4$$

$$-1(-3x + 2y = -4)$$

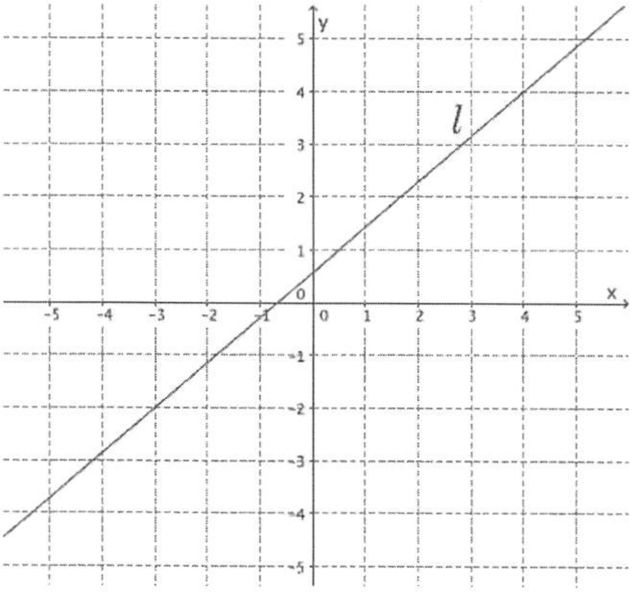
$$3x - 2y = 4$$

I need to multiply each term on both the right and the left sides of the equation by -1 so that a is not negative.

G8-M4-Lesson 21: Some Facts About Graphs of Linear Equations in Two Variables

1. Write the equation for the line l shown in the figure.

I need to identify two points to find the slope. I will use $(-3,-2)$ and $(4,4)$ because they have integer coordinates.



Using the points $(-3,-2)$ and $(4,4)$, the slope of the line is

$$m = \frac{4-(-2)}{4-(-3)}$$
$$= \frac{6}{7}$$

The y-intercept point of the line is

$$4 = \frac{6}{7}(4) + b$$
$$4 = \frac{24}{7} + b$$
$$4 - \frac{24}{7} = \frac{24}{7} - \frac{24}{7} + b$$
$$\frac{4}{7} = b$$

I can see that the line doesn't intersect the y-axis at integer coordinates, so I need to calculate the y-intercept, $(0, b)$. I can use either point to substitute into my equation $y = mx + b$.

The equation of the line is $y = \frac{6}{7}x + \frac{4}{7}$.

2. Write the equation for the line that goes through point $(11, -8)$ with slope $m = 5$.

$$-8 = 5(11) + b$$
$$-8 = 55 + b$$
$$-63 = b$$

I know the slope. I only need to calculate the y-intercept.

The equation of the line is $y = 5x - 63$.

3. Determine the equation of the line that goes through points $(-7,3)$ and $(5,-6)$.

The slope of the line is

$$m = \frac{-6-3}{5-(-7)} = \frac{-9}{12} = -\frac{3}{4}$$

This problem is similar to Problem 1, but without a graph.

The y-intercept point of the line is

$$-6 = -\frac{3}{4}(5) + b$$
$$-6 = -\frac{15}{4} + b$$
$$-\frac{9}{4} = b$$

The equation of the line is $y = -\frac{3}{4}x - \frac{9}{4}$.

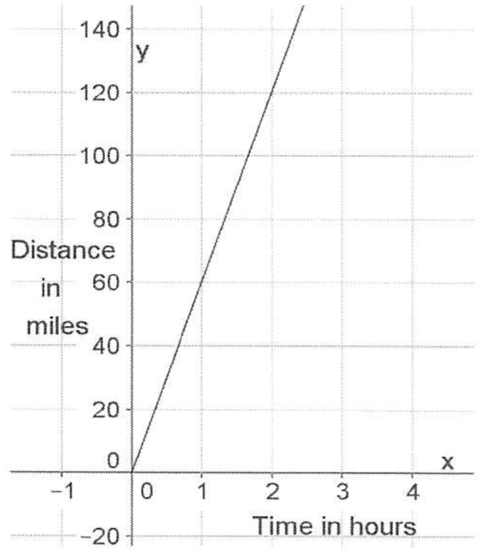
G8-M4-Lesson 22: Constant Rates Revisited

1. Train A can travel a distance of 450 miles in 7 hours.

   a. Assuming the train travels at a constant rate, write the linear equation that represents the situation.

   Let y represent the total number of miles Train A travels in x hours. We can write $\frac{y}{x} = \frac{450}{7}$ and $y = \frac{450}{7}x$.

   b. The figure represents the constant rate of travel for Train B. Which train is faster? Explain.



   To see which train is faster, I need to compare the slopes or rates of change.

   Train A is faster than Train B. The slope, or rate, for Train A is $\frac{450}{7}$, and the slope of the line for Train B is $\frac{60}{1}$. When you compare the slopes, you see that $\frac{450}{7} > 60$.

2. Norton and Sylvia read the same book. Norton can read 33 pages in 8 minutes.

   a. Assuming he reads at a constant rate, write the linear equation that represents the situation.

   Let y represent the total number of pages Norton can read in x minutes. We can write $\frac{y}{x} = \frac{33}{8}$ and $y = \frac{33}{8}x$.

   b. The table of values below represents the number of pages read by Sylvia for a few selected time intervals. Assume Sylvia is reading at a constant rate. Who reads faster? Explain.

Minutes (x)	Pages Read (y)
3	11
5	$\frac{55}{3}$
6	22
8	$\frac{88}{3}$

Since Sylvia is reading at a constant rate, I can use any two points to calculate the slope or rate of change.

Norton reads faster. Using the table of values, I can find the slope that represents Sylvia's constant rate of reading: $\frac{11}{3}$. The slope or rate for Norton is $\frac{33}{8}$. When you compare the slopes, you see that $\frac{33}{8} > \frac{11}{3}$.

G8-M4-Lesson 23: The Defining Equation of a Line

1. Do the equations $3x - 5y = 8$ and $6x - 10y = 16$ define the same line? Explain.

They define the same line when $\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$ is true. In $3x - 5y = 8$, $a = 3, b = -5$, and $c = 8$. In $6x - 10y = 16, a' = 6, b' = -10$, and $c' = 16$.

Yes, these equations define the same line. When you compare the constants from each equation, you get

$$\frac{a'}{a} = \frac{6}{3} = 2, \frac{b'}{b} = \frac{-10}{-5} = 2, \textit{ and } \frac{c'}{c} = \frac{16}{8} = 2.$$

When I multiply the first equation by 2, I get the second equation.

$$(3x - 5y = 8)2$$
$$6x - 10y = 16$$

Therefore, these equations define the same line.

2. Do the equations $y = -\frac{7}{5}x - 4$ and $14x + 10y = -40$ define the same line? Explain.

I need to rewrite the first equation in standard form before I can determine if they define the same line.

Yes, these equations define the same line. When you rewrite the first equation in standard form:

$$y = -\frac{7}{5}x - 4$$
$$\left(y = -\frac{7}{5}x - 4\right)5$$
$$5y = -7x - 20$$
$$7x + 5y = -20.$$

When you compare the constants from each equation:

$$\frac{a'}{a} = \frac{14}{7} = 2, \frac{b'}{b} = \frac{10}{5} = 2, \textit{ and } \frac{c'}{c} = \frac{-40}{-20} = 2.$$

When I multiply the first equation by 2, I get the second equation.

$$(7x + 5y = -20)2$$
$$14x + 10y = -40$$

Therefore, these equations define the same line.

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3. Write an equation that would define the same line as $9x - 12y = 15$.

Answers will vary. When you multiply the equation by 2:

$$(9x - 12y = 15)2$$
$$18x - 24y = 30.$$

When you compare the constants from each equation:

$$\frac{a'}{a} = \frac{18}{9} = 2, \frac{b'}{b} = \frac{-24}{-12} = 2, \text{ and } \frac{c'}{c} = \frac{30}{15} = 2.$$

Therefore, these equations define the same line.

I can multiply the equation by any number other than zero and then make sure that $\frac{a'}{a}$, $\frac{b'}{b}$, $\frac{c'}{c}$ are all equal to the same number.

4. Challenge: Show that if the two lines given by $ax + by = c$ and $a'x + b'y = c'$ are the same when $b = 0$ (vertical lines), then there exists a nonzero number s so that $a' = sa$, $b' = sb$, and $c' = sc$.

When $b = 0$, then $b' = 0$, and the equations are $ax = c$ and $a'x = c'$.

I need to write the equations when $b = 0$. Since the problem said they were the same line, I will solve for x in both equations so that I can use substitution.

We can rewrite the equations as $x = \frac{c}{a}$ and $x = \frac{c'}{a'}$. Because the equations graph as the same line, then we know that

$$\frac{c}{a} = \frac{c'}{a'}$$

and we can rewrite those fractions as

$$\frac{a'}{a} = \frac{c'}{c}.$$

I can use properties of equality to rewrite in the form I need.

These fractions are equal to the same number. Let that number be s. Then $\frac{a'}{a} = s$ and $\frac{c'}{c} = s$. Therefore, $a' = sa$ and $c' = sc$.

G8-M4-Lesson 24: Introduction to Simultaneous Equations

1. Allen and Regina walk at constant speeds. Allen can walk 1 mile in 60 minutes, and Regina can walk 2 miles in 90 minutes. Regina started walking 10 minutes after Allen. Assuming they walk the same path, when will Regina catch up to Allen?

   a. Write the linear equation that represents Regina's constant speed.

   Regina's rate is $\frac{2}{90}$ miles per minute, which is the same as $\frac{1}{45}$ miles per minute. If Regina continues walking y miles in x minutes at a constant speed, then $y = \frac{1}{45}x$.

   Since they are walking at constant speeds, I can write equations using average speed like I did in Lesson 10.

   I need to define my variables for the equations to make sense.

   b. Write the linear equation that represents Allen's constant speed. Make sure to include in your equation the extra time that Allen was able to walk.

   Allen's rate is $\frac{1}{60}$ miles per minute. If Allen continues walking y miles in x minutes at a constant speed, then $y = \frac{1}{60}x$. To account for the extra time that Allen gets to walk, we write the equation

   To account for the extra time, I need to add 10 minutes to Allen's time of x minutes.

   $$y = \frac{1}{60}(x + 10)$$
   $$y = \frac{1}{60}x + \frac{1}{6}$$

   When I distribute $\frac{1}{60}$ to 10, I can write it as $\frac{10}{60}$, or $\frac{1}{6}$.

   c. Write the system of linear equations that represents this situation.

   Writing a system means to write both of the equations with the bracket in front.

   $$\begin{cases} y = \frac{1}{45}x \\ y = \frac{1}{60}x + \frac{1}{6} \end{cases}$$

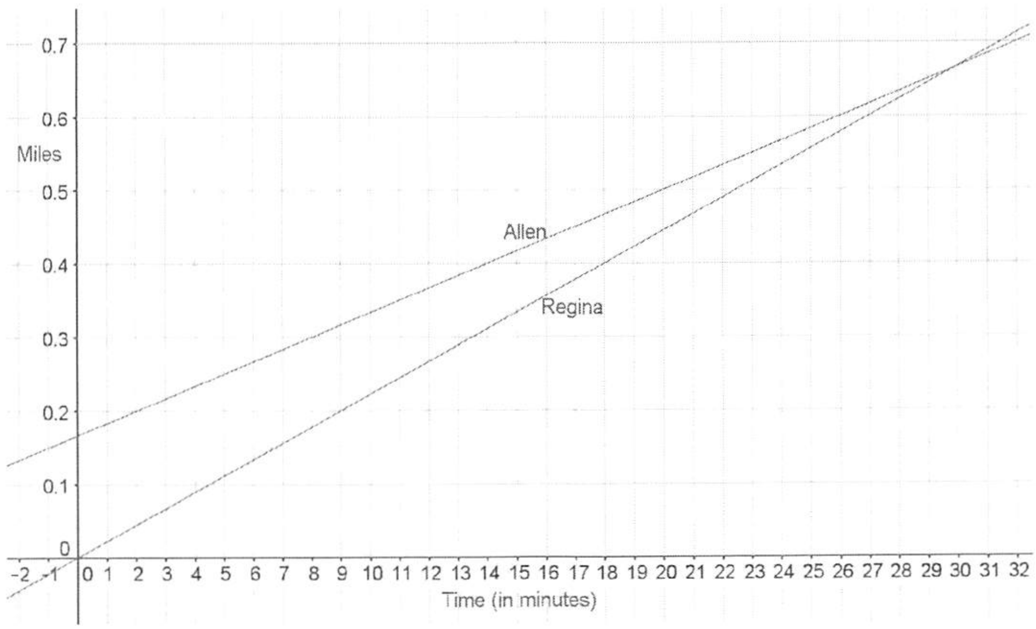
d. Sketch the graphs of the two equations.

I put information about Regina's walk in a table to help me graph.

Number of Minutes (x)	Miles Walked (y)
0	0
9	0.2
18	0.4

I can do the same with information about Allen's walk.

I will label the axis according to how I defined my variables. I need to label the graph of each line.



e. Will Regina ever catch up to Allen? If so, approximately when?

Yes, Regina will catch up to Allen after about 30 *minutes or about* 0.65 *miles.*

f. At approximately what point do the graphs of the lines intersect?

The lines intersect at approximately $(30, 0.65)$.

I can use the graph to see at what point the graphs of the lines intersect. This will tell me when Regina will catch up to Allen.

G8-M4-Lesson 25: Geometric Interpretation of the Solutions of a Linear System

1. Sketch the graphs of the linear system on a coordinate plane: $\begin{cases} y = -\frac{1}{9}x - 3 \\ -2x + 3y = 12 \end{cases}$

I will use the slope and y-intercept to help me graph the equation of this line.

For the equation $y = -\frac{1}{9}x - 3$*:*

The slope is $-\frac{1}{9}$*, and the* y*-intercept is* $(0, -3)$.

Since this equation is in standard form, I will fix $x = 0$ to find y (the y-intercept) and fix $y = 0$ to find x (the x-intercept) to help me graph the line of this equation.

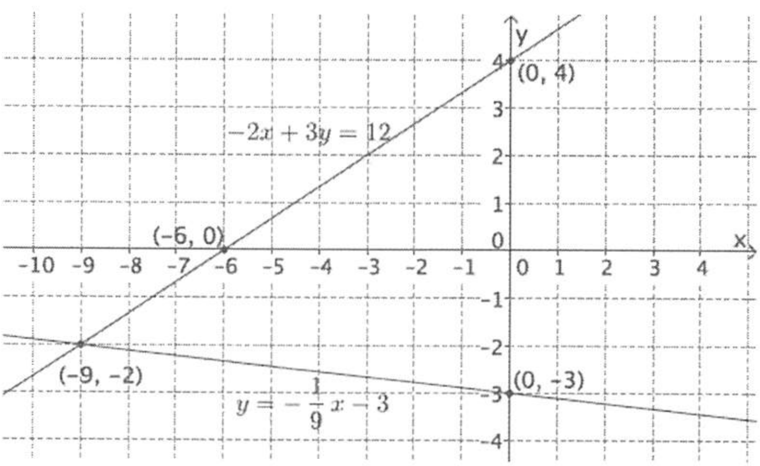
For the equation $-2x + 3y = 12$*:*

$$-2(0) + 3y = 12$$
$$3y = 12$$
$$y = 4$$

The y*-intercept is* $(0, 4)$.

$$-2x + 3(0) = 12$$
$$-2x = 12$$
$$x = -6$$

The x*-intercept is* $(-6, 0)$.



To locate where the graphs of the lines intersect, I will use graph paper so that I can be as accurate as possible.

a. Name the ordered pair where the graphs of the two linear equations intersect.

$(-9, -2)$

b. Verify that the ordered pair named in part (a) is a solution to $y = -\frac{1}{9}x - 3$.

$$-2 = -\frac{1}{9}(-9) - 3$$
$$-2 = 1 - 3$$
$$-2 = -2$$

The left and right sides of the equation are equal.

c. Verify that the ordered pair named in part (a) is a solution to $-2x + 3y = 12$.

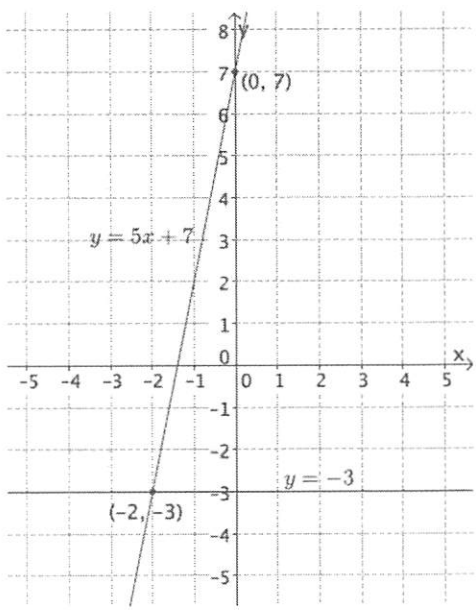
$$-2(-9) + 3(-2) = 12$$
$$18 - 6 = 12$$
$$12 = 12$$

The left and right sides of the equation are equal.

If the solutions I found on the graph of the lines satisfies both equations, then I have found an ordered pair of the system.

2. Sketch the graphs of the linear system on a coordinate plane: $\begin{cases} y = 5x + 7 \\ y = -3 \end{cases}$

The equation of the form $y = c$, where c is a constant, is a horizontal line passing though the point $(0, c)$. I know that -3 will be the y-coordinate of my intersection point.



I can write the slope of 5 with the denominator of 1 as the fraction $\frac{5}{1}$.

For the equation $y = 5x + 7$:

The slope is $\frac{5}{1}$, and the y-intercept is $(0, 7)$.

I need to verify that the ordered pair is a solution to both of the equations.

Name the ordered pair where the graphs of the two linear equations intersect.

$(-2, -3)$

G8-M4-Lesson 26: Characterization of Parallel Lines

I need to determine if the graphs of the lines are parallel. Parallel lines do not intersect, which means parallel lines have no solution.

Answer Problems 1–2 without graphing the equations.

1. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} 2x + 5y = 9 \\ -4x - 10y = 4 \end{cases}$$

Standard form is $ax + by = c$, where a, b, c are constants and a and b are not both zero.

I learned from Lesson 23 that when equations are written in standard form, I know the slope is $m = -\frac{a}{b}$, and the y-intercept is $\frac{c}{b}$.

No, this system does not have a solution. The slope of the first equation is $-\frac{2}{5}$, and the slope of the second equation is $-\frac{4}{10}$, which is equivalent to $-\frac{2}{5}$. Since the slopes are the same and the lines are distinct, these equations will graph as parallel lines. Parallel lines never intersect, which means this system has no solution.

2. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} \frac{7}{4}x + 2 = y \\ x + 2y = 4 \end{cases}$$

If the slopes are different, these equations will graph as non-parallel lines and intersect at some point. That means they will have a solution.

The first equation is written in slope-intercept form. The slope is $\frac{7}{4}$.

Yes, this system does have a solution. The slope of the first equation is $\frac{7}{4}$, and the slope of the second equation is $-\frac{1}{2}$. Since the slopes are different, these equations will graph as non-parallel lines, which means they will intersect at some point.

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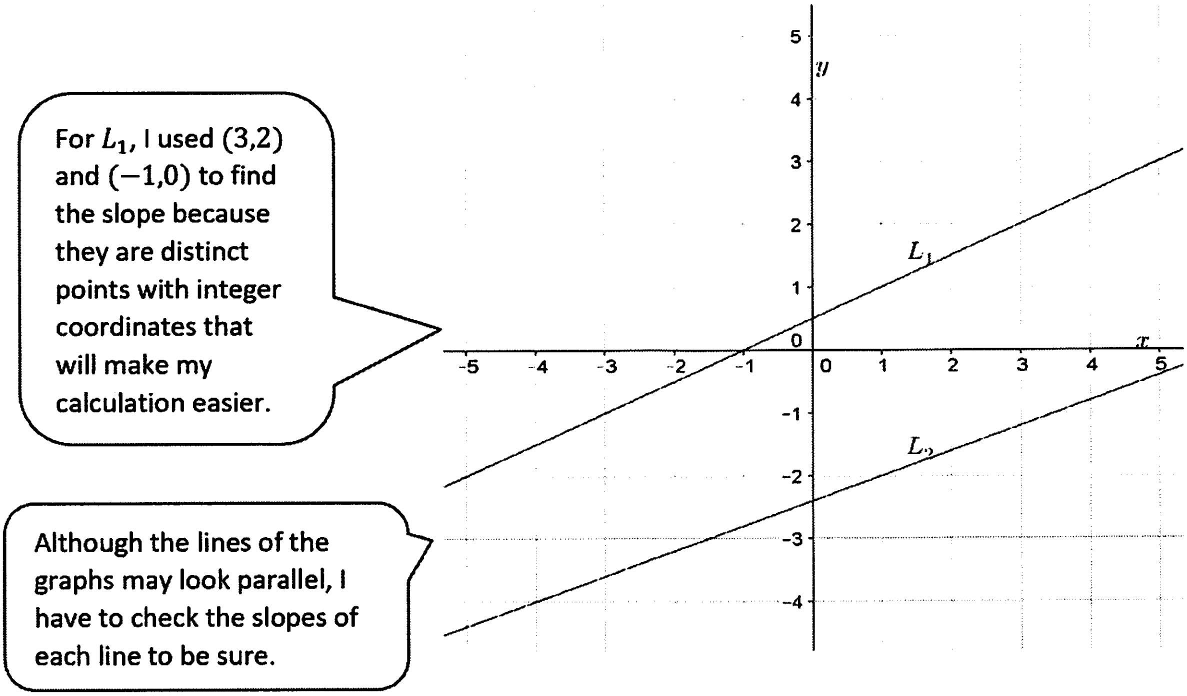
3. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



The slope of L_1 is $\frac{1}{2}$, and the slope of L_2 is $\frac{2}{5}$. Since the slopes are different, these lines are nonparallel lines, which means they will intersect at some point. Therefore, the system of linear equations whose graphs are the given lines will have a solution.

G8-M4-Lesson 27: Nature of Solutions of a System of Linear Equations

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

1. $\begin{cases} y = -\frac{4}{5}x + 9 \\ 4x + 5y = 9 \end{cases}$

If the equations have the same slope and different y-intercepts, then the equations graph as parallel lines, which means the system doesn't have a solution.

The slopes of these two equations are the same, and the y-intercepts are different, which means they graph as parallel lines. Therefore, this system will have no solutions.

2. $\begin{cases} 2x - 3y = 12 \\ y = \frac{2}{3}x - 4 \end{cases}$

I notice that if I multiply the second equation by 3, the result is $3y = 2x - 12$. When I use my properties of equality, I see the second equation is the same as the first. This means that I have the same line; therefore, I have infinitely many solutions.

These equations define the same line. Therefore, this system will have infinitely many solutions.

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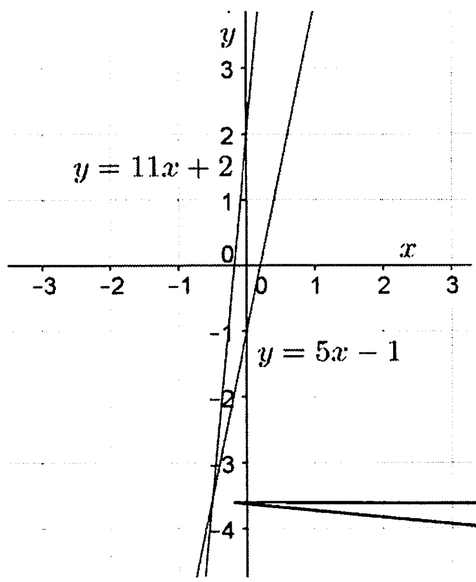
3. $\begin{cases} y = 5x - 1 \\ y = 11x + 2 \end{cases}$

Since the equations are both equal to y, I can use substitution, and write the equations equal to each other and solve for x.

$$5x - 1 = 11x + 2$$
$$-3 = 6x$$
$$-\frac{1}{2} = x$$

Once I solve for x, then I can use substitution again in either equation and solve for y.



$$y = 5\left(-\frac{1}{2}\right) - 1$$
$$y = -\frac{5}{2} - 1$$
$$y = -\frac{7}{2}$$

The solution is $\left(-\frac{1}{2}, -\frac{7}{2}\right)$.

I see that the graphs of the lines intersect at $\left(-\frac{1}{2}, -\frac{7}{2}\right)$.

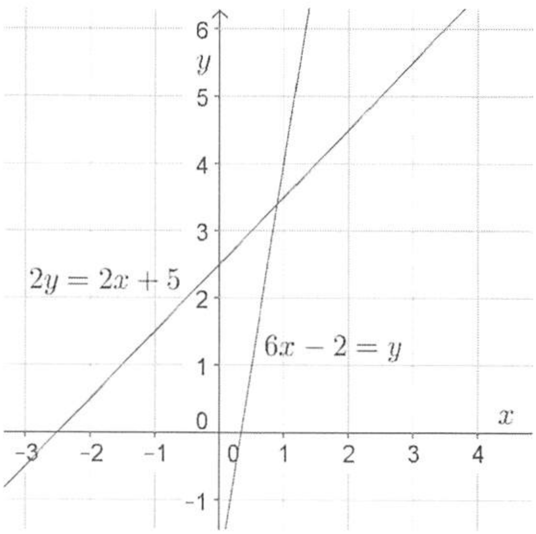
4. $\begin{cases} 6x - 2 = y \\ 2y = 2x + 5 \end{cases}$

I can multiply the first equation by 2 to produce an equivalent equation, namely $12x - 4 = 2y$. Now that both equations are equal to $2y$, the expressions $12x - 4$ and $2x + 5$ can be written equal to one another.

$$(6x - 2 = y)2$$
$$12x - 4 = 2y$$

I can write the system as $\begin{cases} 12x - 4 = 2y \\ 2y = 2x + 5. \end{cases}$

$$\begin{cases} 12x - 4 = 2y \\ 2y = 2x + 5 \end{cases}$$
$$12x - 4 = 2x + 5$$
$$10x = 9$$
$$x = \frac{9}{10}$$



$$6\left(\frac{9}{10}\right) - 2 = y$$
$$\frac{54}{10} - 2 = y$$
$$\frac{17}{5} = y$$

The solution is $\left(\frac{9}{10}, \frac{17}{5}\right)$.

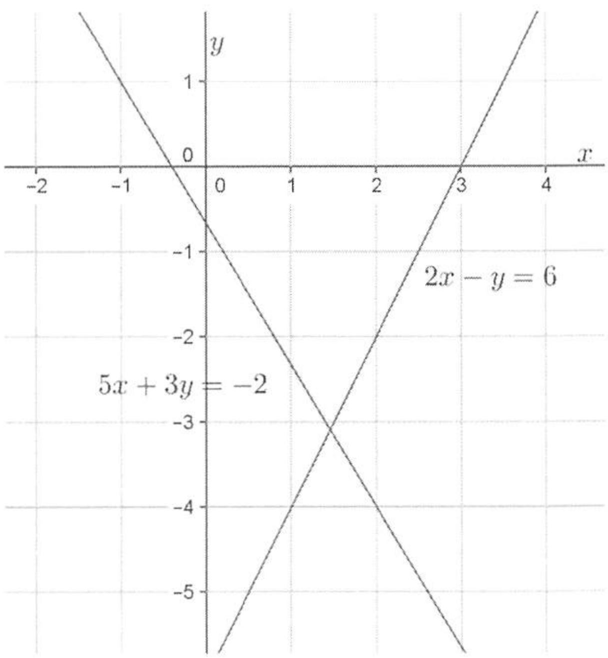
G8-M4-Lesson 28: Another Computational Method of Solving a Linear System

I can determine if the solutions exist by checking the slope and the y-intercepts like I did in the previous lesson's Problem Set.

Determine the solution, if it exists, for each system of linear equations. Verify your solution on the coordinate plane.

1. $\begin{cases} 5x + 3y = -2 \\ 2x - y = 6 \end{cases}$

If I multiply the second equation by 3, then I will eliminate x and be able to solve for y.



$$3(2x - y = 6)$$
$$6x - 3y = 18$$
$$\begin{cases} 5x + 3y = -2 \\ 6x - 3y = 18 \end{cases}$$
$$5x + 3y + 6x - 3y = -2 + 18$$
$$11x = 16$$
$$x = \frac{16}{11}$$

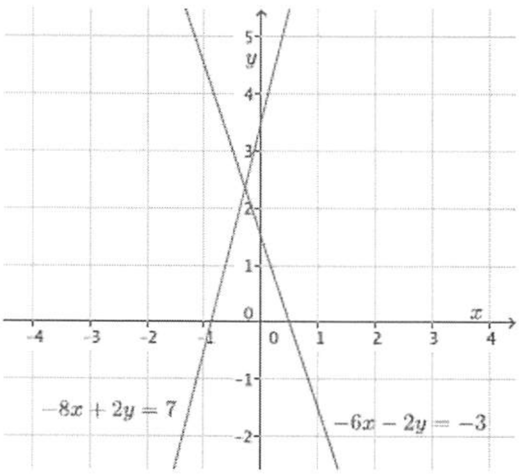
$$2\left(\frac{16}{11}\right) - y = 6$$
$$-y = \frac{34}{11}$$
$$y = -\frac{34}{11}$$

The solution is $\left(\frac{16}{11}, -\frac{34}{11}\right)$.

2. $\begin{cases} -6x - 2y = -3 \\ -8x + 2y = 7 \end{cases}$

I notice that since the first equation has $-2y$ and the second equation has $+2y$, when I add the equations together, y will be eliminated, and I can solve for x first.

$$-6x - 2y - 8x + 2y = -3 + 7$$
$$-14x = 4$$
$$x = -\frac{4}{14}$$

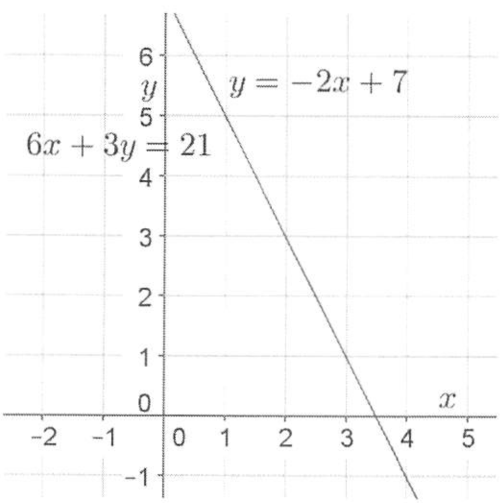


$$-6\left(-\frac{4}{14}\right) - 2y = -3$$
$$\frac{12}{7} - 2y = -3$$
$$-2y = -\frac{33}{7}$$
$$y = \frac{33}{14}$$

The solution is $\left(-\frac{4}{14}, \frac{33}{14}\right)$.

3. $\begin{cases} y = -2x + 7 \\ 6x + 3y = 21 \end{cases}$

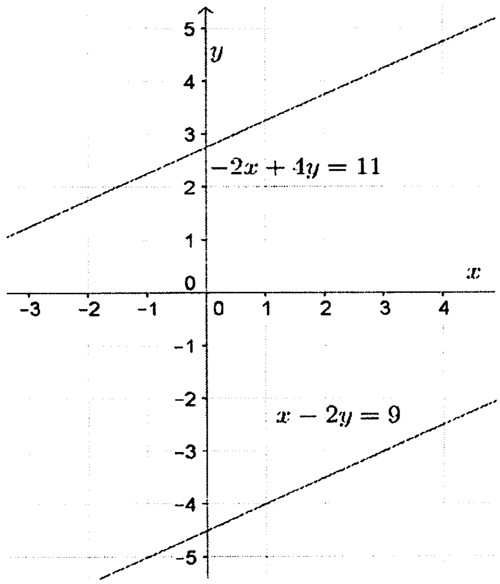
When I substituted for y with the first equation into the second equation, $6x + 3(-2x + 7) = 21$, it resulted in an identity, namely $21 = 21$. This means the two equations will graph the same line.



These equations define the same line. Therefore, this system will have infinitely many solutions.

4. $\begin{cases} -2x + 4y = 11 \\ x - 2y = 9 \end{cases}$

When I multiplied the second equation by 2, I eliminated both x and y. The result was an untrue statement, namely $0 \neq 29$. I should check the slopes and y-intercepts.



The equations graph as distinct lines. The slopes of these two equations are the same, and the y-intercepts are different, which means they graph as parallel lines. Therefore, this system will have no solution.

G8-M4-Lesson 29: Word Problems

1. Two numbers have a sum of 853 and a difference of 229. What are the two numbers?

Let x represent one number and y represent the other number.

$$\begin{cases} x+y=853 \\ x-y=229 \end{cases}$$

$$x+y+x-y=853+229$$

$$2x=1082$$

$$x=541$$

$$541+y=853$$

$$y=312$$

I can check my answer mentally.

Sum means I add the two numbers together, and difference means I subtract one number from the other. Since I don't know either number, I need to define my variables with two different letters.

The solution is $(541, 312)$. The two numbers are 541 and 312.

2. The sum of the ages of two sisters is 36. The younger sister is 6 more than a fifth of the older sister's age. How old is each sister?

Let x represent the age of the younger sister and y represent the age of the older sister.

A fifth of the older sister's age means to multiply $\frac{1}{5}$ times y, the age of the older sister.

$$\begin{cases} x+y=36 \\ x=6+\frac{1}{5}y \end{cases}$$

I will use the substitution method since x is isolated. I will replace x with $6+\frac{1}{5}y$ in the first equation.

$$6+\frac{1}{5}y+y=36$$

$$6+\frac{6}{5}y=36$$

$$\frac{6}{5}y=30$$

$$y=25$$

$$x+25=36$$

$$x=11$$

If I let x represent the age of the older sister and y represent the age of the younger sister, the second equation would be $y=6+\frac{1}{5}x$. I would use the same method to solve.

Check:

$$11=6+\frac{1}{5}(25)$$

$$11=6+5$$

$$11=11$$

The solution is $(11, 25)$. The older sister is 25 years old, and the younger sister is 11 years old.

3. Some friends went to the local movie theater and bought three buckets of large popcorn and four boxes of candy. The total for the snacks was \$30.50. The last time you were at the theater, you bought a large popcorn and two boxes of candy, and the total was \$12.50. How much would 2 large buckets of popcorn and 3 boxes of candy cost?

Let x represent the cost of a large bucket of popcorn and y represent the cost of a box of candy.

$$\begin{cases} 3x + 4y = 30.50 \\ x + 2y = 12.50 \end{cases}$$

I have choices. I could eliminate x by multiplying the second equation by -3 or eliminate y by multiplying the second equation by -2.

$$-2(x + 2y = 12.50)$$
$$-2x - 4y = -25$$
$$\begin{cases} 3x + 4y = 30.50 \\ -2x - 4y = -25 \end{cases}$$
$$3x + 4y - 2x - 4y = 30.50 - 25$$
$$3x - 2x = 5.50$$
$$x = 5.50$$

The question is asking about the *cost* of items, not the number of items. I need to define my variables as the *cost* of each item.

$$5.50 + 2y = 12.50$$
$$2y = 7$$
$$y = 3.50$$

The solution is $(5.50, 3.50)$.

Check:

$$3(5.50) + 4(3.50) = 30.50$$
$$16.50 + 14 = 30.50$$
$$30.50 = 30.50$$

Once I find out the cost of each item, I can determine the cost for 2 large popcorns and 3 boxes of candy.

Since a large bucket of popcorn costs $\$5.50$ *and a box of candy costs* $\$3.50$*, then the equation to find the cost of two large buckets of popcorn and three boxes of candy is* $2(5.50) + 3(3.50) = 11 + 10.50$*, which is equal to* 21.50*. Therefore, the cost of two large buckets of popcorn and three boxes of candy is* $\$21.50$.

G8-M4-Lesson 30: Conversion Between Celsius and Fahrenheit

1. Does the equation $t°\mathrm{C} = (32 + 1.8t)°\mathrm{F}$ work for any rational number t? Check that it does with $t = 12\frac{1}{5}$ and $t = -12\frac{1}{5}$.

I will use substitution with $t = 12\frac{1}{5}$ and $t = -12\frac{1}{5}$.

$$\left(12\frac{1}{5}\right)°\mathrm{C} = \left(32 + 1.8 \times 12\frac{1}{5}\right)°\mathrm{F} = (32 + 21.96)°\mathrm{F} = 53.96°\mathrm{F}$$

This means that $12\frac{1}{5}°\mathrm{C}$ is the same as $53.96°\mathrm{F}$.

$$\left(-12\frac{1}{5}\right)°\mathrm{C} = \left(32 + 1.8 \times \left(-12\frac{1}{5}\right)\right)°\mathrm{F} = (32 - 21.96)°\mathrm{F} = 10.04°\mathrm{F}$$

2. Knowing that $t°\mathrm{C} = \left(32 + \frac{9}{5}t\right)°\mathrm{F}$ for any rational number t, show that for any rational number d, $d°\mathrm{F} = \left(\frac{5}{9}(d - 32)\right)°\mathrm{C}$.

I will write down everything I know from the problem and lesson.

From the lesson, I know that $d°\mathrm{F} = \left(32 + \frac{9}{5}t\right)°\mathrm{F}$.

That implies that $d = \left(32 + \frac{9}{5}t\right)$.

From the problem, I know that $t°\mathrm{C} = \left(32 + \frac{9}{5}t\right)°\mathrm{F}$.

From the lesson, I know that $t°\mathrm{C} = d°\mathrm{F}$.

I will use these equations to help me show that $d°\mathrm{F} = \left(\frac{5}{9}(d - 32)\right)°\mathrm{C}$.

I will start by solving for t.

EUREKA MATH

Since $d°\mathrm{F}$ *can be found by* $\left(32 + \frac{9}{5}t\right)$, *then* $d = \left(32 + \frac{9}{5}t\right)$, *and* $d°\mathrm{F} = t°\mathrm{C}$. *Substituting* $d = \left(32 + \frac{9}{5}t\right)$ *into* $d°\mathrm{F}$ *we get*

$$d°\mathrm{F} = \left(32 + \frac{9}{5}t\right)°\mathrm{F}$$
$$d = 32 + \frac{9}{5}t$$
$$d - 32 = \frac{9}{5}t$$
$$\frac{5}{9}(d - 32) = t$$

Now that we know $t = \frac{5}{9}(d - 32)$, *then* $d°\mathrm{F} = \left(\frac{5}{9}(d - 32)\right)°\mathrm{C}$.

Once I know t, I can substitute into $t°\mathrm{C} = d°\mathrm{F}$ to show that for any rational number d, $d°\mathrm{F} = \left(\frac{5}{9}(d - 32)\right)°\mathrm{C}$.

G8-M4-Lesson 31: System of Equations Leading to Pythagorean Triples

Lesson Notes

Any three numbers, a, b, and c, that satisfy $a^2 + b^2 = c^2$ are considered a triple. A Pythagorean triple is a set of three *whole numbers*, a, b, and c, that satisfy the equation $a^2 + b^2 = c^2$.

Examples

1. Identify a Pythagorean triple (numbers that satisfy $a^2 + b^2 = c^2$), using the known Pythagorean triple 5, 12, 13.

   Answers will vary.

   A triple is $10, 24, 26$. *I found these by multiplying each of* 5, 12, *and* 13 *by* 2.

   I need to multiply the known triple by a whole number to ensure that I produce a Pythagorean triple.

2. Identify a triple (numbers that satisfy $a^2 + b^2 = c^2$), using the known Pythagorean triple 5, 12, 13.

   Answers will vary.

   A triple is $3.5, 8.4, 9.1$. *I found these by multiplying each of* 5, 12, *and* 13 *by* 0.7.

   To produce a triple, I will multiply each number in the known triple by a number between 0 and 1.

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3. Use the system $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$ to find Pythagorean triples for the given values of s and t. Recall that the solution, in the form of $\left(\frac{c}{b}, \frac{a}{b}\right)$, is the triple, a, b, c.

$s = 2, t = 5$

This system will produce triples only if $t > s$.

I will use elimination to solve the system by summing two equations.

$$\begin{cases} x + y = \frac{5}{2} \\ x - y = \frac{2}{5} \end{cases}$$

$$x + y + x - y = \frac{5}{2} + \frac{2}{5}$$
$$2x = \frac{29}{10}$$
$$x = \frac{29}{20}$$

$$\frac{29}{20} + y = \frac{5}{2}$$
$$y = \frac{5}{2} - \frac{29}{20}$$
$$y = \frac{21}{20}$$

Then the solution is $\left(\frac{29}{20}, \frac{21}{20}\right)$***, and the triple is*** $20, 21, 29$.

I write the numerators and denominator in ascending order. The denominator of both equations is b, the smaller numerator is a, and the larger numerator is c.

4. Use a calculator to verify that you found a Pythagorean triple in Problem 2. Show your work below.

For the triple $20, 21, 29$***:***

The longest side of the right triangle is the hypotenuse identified by c.

$$20^2 + 21^2 = 29^2$$
$$400 + 441 = 841$$
$$841 = 841$$

If the triple I found in Problem 2 cannot be verified, I will need to go back and check my work.