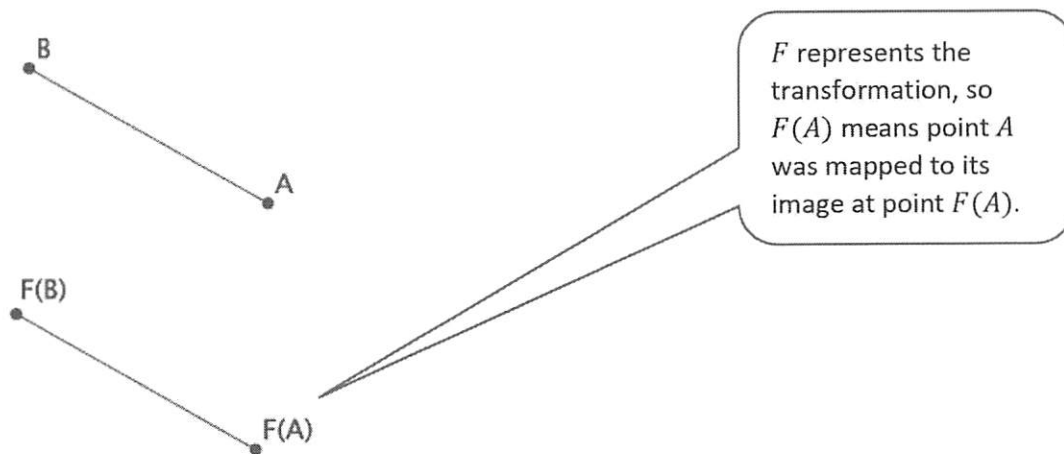# Homework Helpers

Grade 8
Module 2

G8-M2-Lesson 1: Why Move Things Around?

Lesson Notes

Transformations of the plane (i.e., translations, reflections, and rotations) are introduced. Transformations are distance preserving.

Example

Using as much of the new vocabulary as you can, try to describe what you see in the diagram below.

B

A

F(B)

F(A)

> F represents the transformation, so $F(A)$ means point A was mapped to its image at point $F(A)$.

There was a transformation, F, that moved point A to its image $F(A)$ and point B to its image $F(B)$. Since a transformation preserves distance, the distance between points A and B is the same as the distance between points $F(A)$ and $F(B)$.

# G8-M2-Lesson 2: Definition of Translation and Three Basic Properties

Lesson Notes

Translations move figures along a vector. The vector has a starting point and an endpoint. Translations map lines to lines, rays to rays, segments to segments, and angles to angles. Translations preserve lengths of segments and degrees of angles.

Examples

1. Use your transparency to translate the angle of 32 degrees, a segment with length 1.5 in., a point, and a circle with radius 2 cm along vector $\overrightarrow{AB}$. Label points and measures (measurements do not need to be precise, but your figure must be labeled correctly). Sketch the images of the translated figures, and label them.

    Note: The figures below have not been drawn to scale.
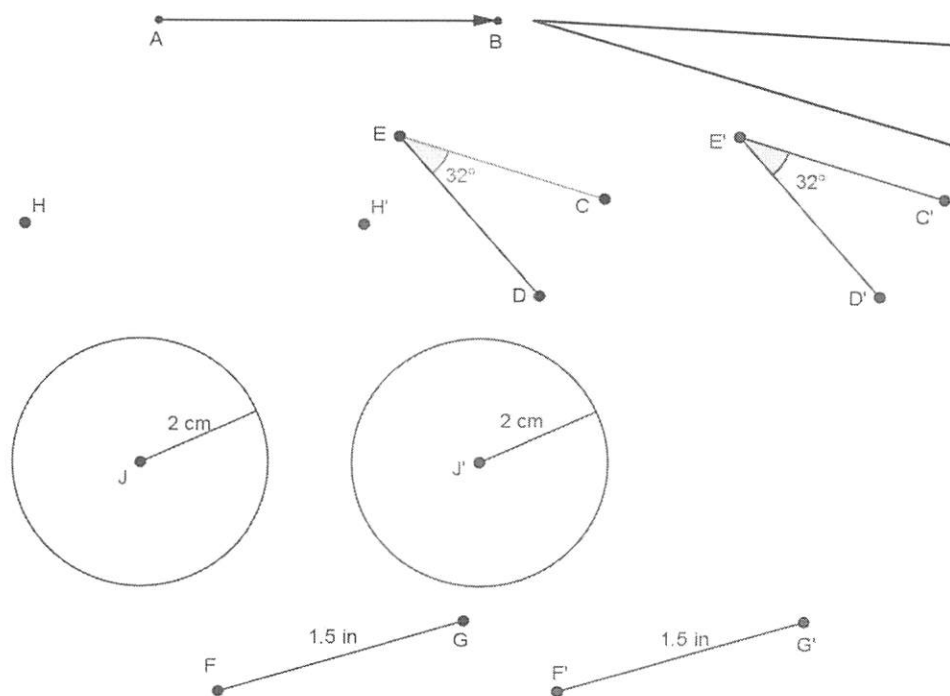
I need to trace the vector and the other figures onto my transparency. I need to slide my transparency along the vector from point A to point B.

Lesson 2:   Definition of Translation and Three Basic Properties

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Use your drawing from Problem 1 to answer the questions below.

2. What is the length of the translated segment? How does this length compare to the length of the original segment? Explain.

    1.5 inches. The length is the same as the original because translations preserve the lengths of segments.

3. What is the length of the radius in the translated circle? How does this radius length compare to the radius of the original circle? Explain.

    2 centimeters. The length is the same as the original because translations preserve the lengths of segments.

4. What is the degree of the translated angle? How does this degree compare to the degree of the original angle? Explain.

    32 degrees. The angles will have the same measure because translations preserve degrees of angles.
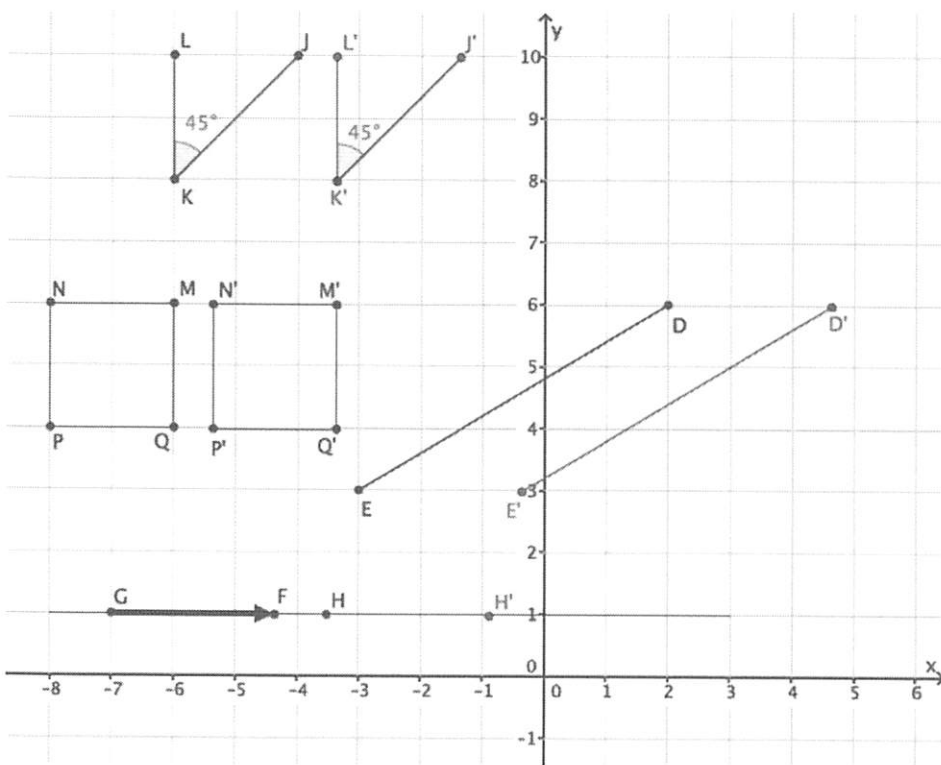
G8-M2-Lesson 3: Translating Lines

Lesson Notes

When lines are translated, they are either parallel to the given line, or they coincide. Translations map parallel lines to parallel lines.

Examples

1. Translate $\angle JKL$, segment DE, point H, and square $MNPQ$ along vector $\overrightarrow{GF}$. Sketch the images, and label all points using prime notation.

> I can use a transparency to translate the images.

2. What is the measure of the translated image of $\angle JKL$? How do you know?

The measure is $45°$. *Translations preserve angle measure.*

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3. Connect D to D'. What do you know about the line that contains the segment formed by connecting points D and D' and the line containing the vector $\overrightarrow{GF}$?

   $\overleftrightarrow{DD'} \parallel \overrightarrow{GF}$

4. Connect H to H'. What do you know about the line that contains the segment formed by connecting points H and H' and the line containing the vector $\overrightarrow{GF}$?

   $\overleftrightarrow{HH}$ and $\overrightarrow{GF}$ coincide.

5. Given that figure $MNPQ$ is a square, what do you know about lines NP and MQ and their translated images? Explain.

   By definition of a square, I know that $\overleftrightarrow{NP} \parallel \overleftrightarrow{MQ}$. Since translations map parallel lines to parallel lines, then $\overleftrightarrow{N'P'} \parallel \overleftrightarrow{M'Q'}$.

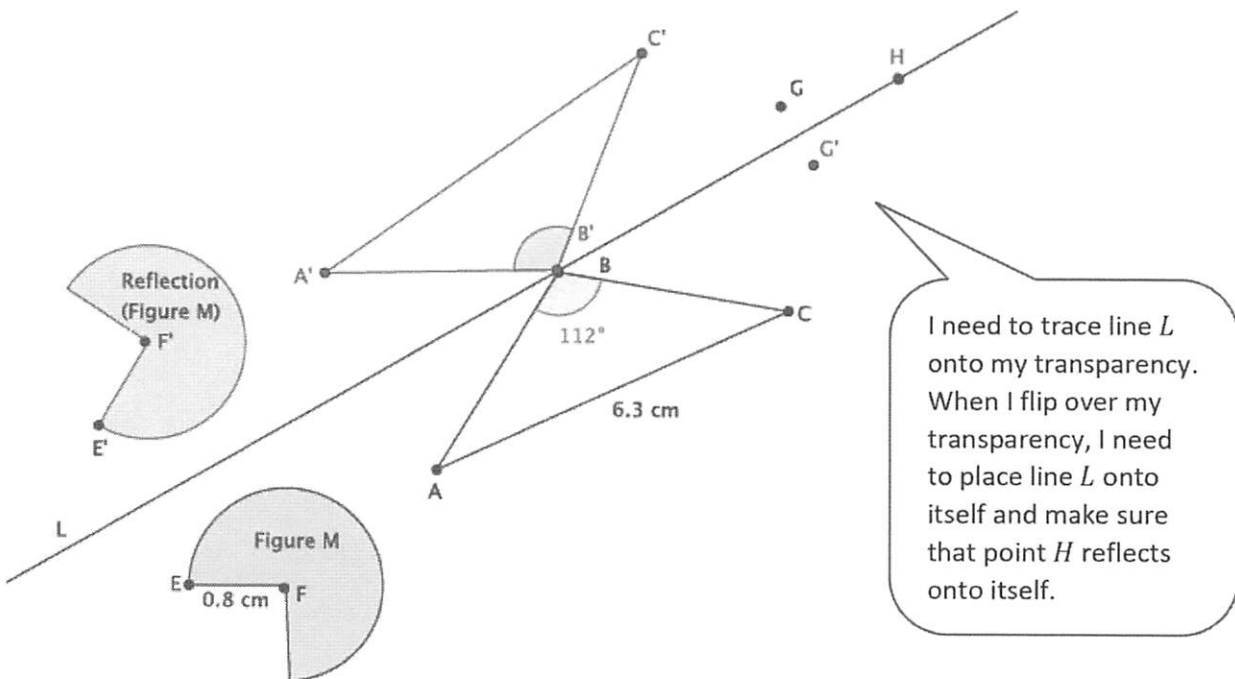   I remember that a square has opposite sides that are parallel.

G8-M2-Lesson 4: Definition of Reflection and Basic Properties

Lesson Notes

Reflections are a basic rigid motion that maps lines to lines, rays to rays, segments to segments, and angles to angles. Basic rigid motions preserve lengths of segments and degrees of measures of angles. Reflections occur across a line called the line of reflection.

Examples

1. In the diagram below, $\angle ABC = 112°$, $AC = 6.3$ cm, $EF = 0.8$ cm, point H is on line L, and point G is off of line L. Let there be a reflection across line L. Reflect and label each of the figures, and answer the questions that follow.

I need to trace line L onto my transparency. When I flip over my transparency, I need to place line L onto itself and make sure that point H reflects onto itself.

Note: Diagram not to scale.

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2. What is the measure of $Reflection(\angle ABC)$? Explain.

    The measure of $Reflection(\angle ABC)$ is $112°$. Reflections preserve degrees of angles.

3. What is the length of $Reflection(EF)$? Explain.

    The length of $Reflection(EF)$ is 0.8 cm. Reflections preserve lengths of segments.

4. What is the length of $Reflection(AC)$?

    The length of $Reflection(AC)$ is 6.3 cm.

5. Three figures in the picture were not moved under the reflection.
    Name the three figures, and explain why they were not moved.

    *Point B, point H, and line L were not moved. All of the points
    that make up the line of reflection remain in the same location
    when reflected. Since points B and H are on the line of reflection,
    they were not moved.*

    I remember my teacher telling me that the line of reflection is reflected, but it isn't moved to a new location.

6. Connect points G and G'. Name the point of intersection of the segment with the line of reflection point Q. What do you know about the lengths of segments QG and QG'?

    Segments QG and QG' are equal in length. The segment GG' connects point G to its image, G'. The line of reflection will go through the midpoint of, or bisect, the segment created when you connect a point to its image.

# G8-M2-Lesson 5: Definition of Rotation and Basic Properties

Lesson Notes

Rotations are a basic rigid motion that maps lines to lines, rays to rays, segments to segments, and angles to angles. Rotations preserve lengths of segments and degrees of measures of angles. Rotations require information about the center of rotation and the degree in which to rotate. Positive degrees of rotation move the figure in a counterclockwise direction. Negative degrees of rotation move the figure in a clockwise direction.

Examples

1. Let there be a rotation by 90° around the center O.
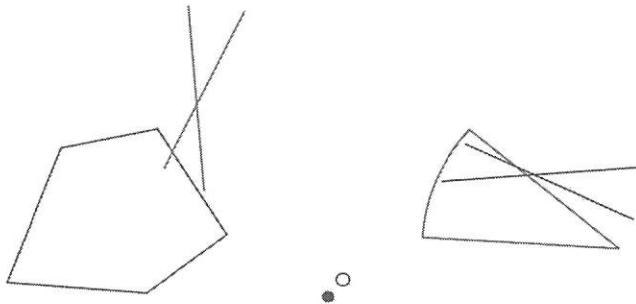
I can use my transparency to help me rotate the figures. I need to remember that rotations by a positive number means to move the figure in the counterclockwise direction.
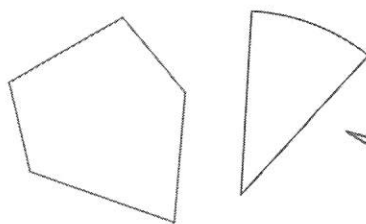
This is the original triangular figure, and the image of it is just above. I know this because the degree of rotation is positive (so figures will move counterclockwise).

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2. A segment of length 18 in. has been rotated d degrees around a center O.
    What is the length of the rotated segment? How do you know?

    The rotated segment will be 18 in. in length. (Rotation 2) states that
    rotations preserve lengths of segments, so the length of the rotated
    segment will remain the same as the original.

I need to remember
that it doesn't matter
how many degrees I
rotate, the basic
properties will be
true. I can find the
numbered Basic
Properties of
Rotation in my
Lesson Summary
box.

3. An angle of size 52° has been rotated d degrees around a center O.
    What is the size of the rotated angle? How do you know?

    The rotated angle will be 52°. (Rotation 3) states that rotations
    preserve the degrees of angles, so the rotated angle will be the same
    size as the original.

G8-M2-Lesson 6: Rotations of 180 Degrees

Lesson Notes

When a line is rotated 180° around a point not on the line, it maps to a line parallel to the given line. A point P with a rotation of 180° around a center O produces a point P' so that P, O, and P' are collinear. When we rotate coordinates 180° around O, the point with coordinates (a, b) is moved to the point with coordinates $(-a, -b)$.

Example

Use the following diagram for Problems 1–5. Use your transparency as needed.

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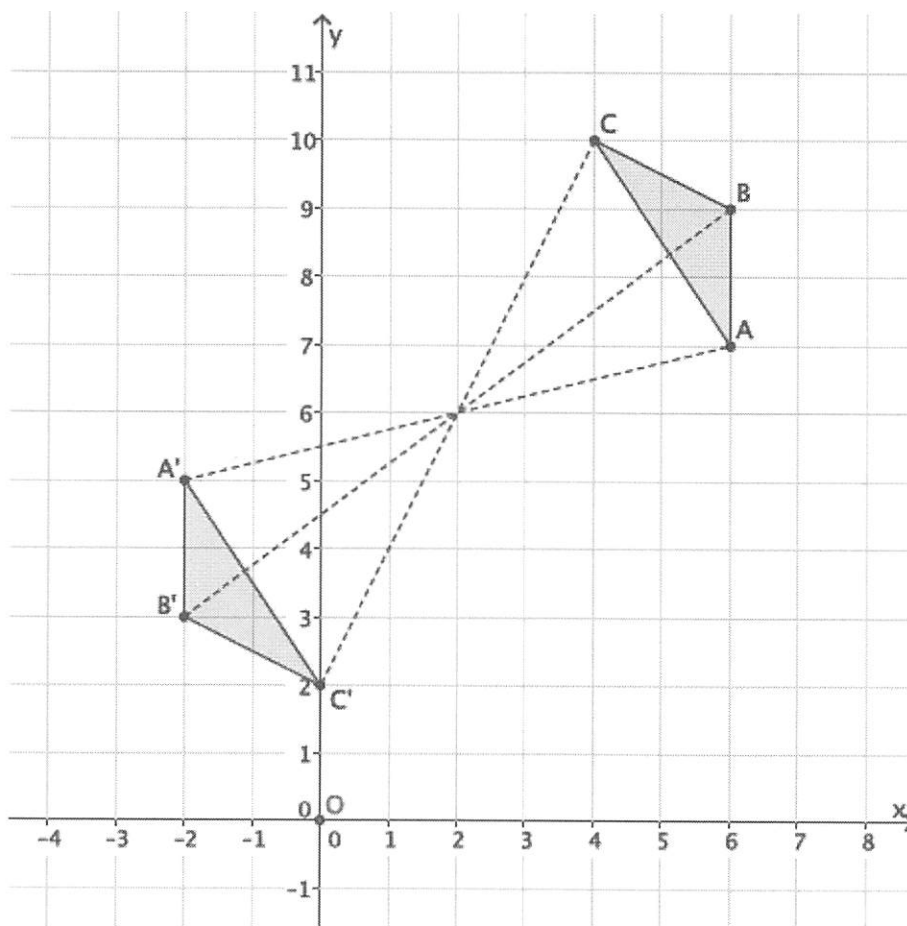
1. Looking only at segment BC, is it possible that a 180° rotation would map segment BC onto segment $B'C'$? Why or why not?

   It is possible because the segments are parallel.

   > I will use my transparency to verify that the segments are parallel. I think the center of rotation is the point $(2, 6)$.

2. Looking only at segment AB, is it possible that a 180° rotation would map segment AB onto segment $A'B'$? Why or why not?

   It is possible because the segments are parallel.

3. Looking only at segment AC, is it possible that a 180° rotation would map segment AC onto segment $A'C'$? Why or why not?

   It is possible because the segments are parallel.

4. Connect point B to point B', point C to point C', and point A to point A'. What do you notice? What do you think that point is?

   All of the lines intersect at one point. The point is the center of rotation. I checked by using my transparency.

   > I checked each segment and its rotated segment to see if they were parallel. I found the center of rotation, so I can say there is a rotation of 180° about a center.

5. Would a rotation map △ ABC onto △ $A'B'C'$? If so, define the rotation (i.e., degree and center). If not, explain why not.

   *Let there be a rotation of 180° around point $(2, 6)$. Then, $Rotation(△ ABC) = △ A'B'C'$.*

G8-M2-Lesson 7: Sequencing Translations

Lesson Notes

Sequences of translations have the same properties of a single translation (i.e., map lines to lines, rays to rays, segments to segments, and angles to angles). Sequences of translations preserve lengths of segments and degrees of measures of angles. If a figure undergoes two transformations and is in the same place as it was originally, then the figure has been mapped onto itself.

Examples

1. Sequence translations of rectangle $ABCD$ (a quadrilateral in which both pairs of opposite sides are parallel) along vectors $\overrightarrow{EF}$ and $\overrightarrow{GH}$. Label the translated images.
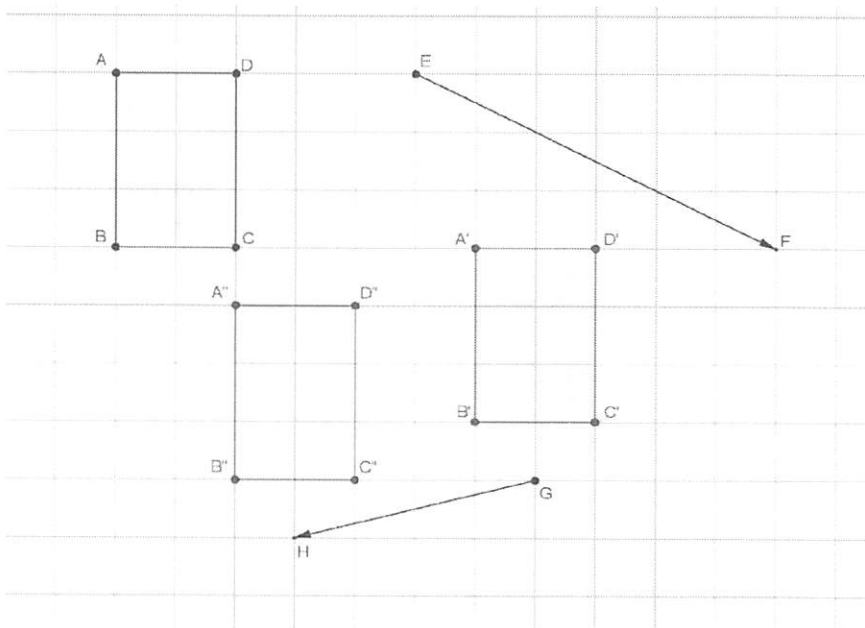
> I will trace the rectangle and the vector $\overrightarrow{EF}$ onto my transparency first. Then I will note the image as $A'B'C'D'$. After I have translated the rectangle along vector $\overrightarrow{EF}$, I will trace vector $\overrightarrow{GH}$ and translate the rectangle $A'B'C'D'$, resulting in the final image $A''B''C''D''$.

2. What do you know about $\overline{AD}$ and $\overline{BC}$ compared with $\overline{A'D'}$ and $\overline{B'C'}$? Explain.
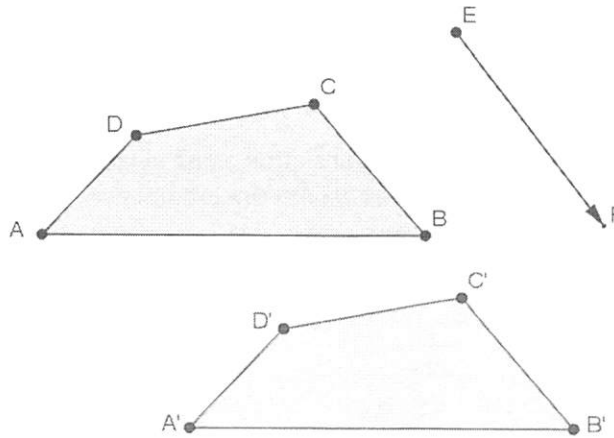
    By the definition of a rectangle, $\overline{AD} \parallel \overline{BC}$. Since translations map parallel lines to parallel lines, I know that $\overline{A'D'} \parallel \overline{B'C'}$.

    > I remember this from Lesson 3.

3. Are the segments $A'B'$ and $A''B''$ equal in length? How do you know?

    Yes, $|A'B'| = |A''B'|$. Translations preserve lengths of segments.

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4. Translate the shape $ABCD$ along the given vector. Label the image.

> I will trace the shape and the vector $\overrightarrow{EF}$ onto my transparency and then note the image as $A'B'C'D'$.

5. What vector would map the shape $A'B'C'D'$ back onto shape $ABCD$?

*Translating the image along vector $\overrightarrow{FE}$ would map the image back onto its original position.*

> Using the same transparency for Problem 4, I will translate along the vector $\overrightarrow{FE}$ to map $A'B'C'D'$ back to the shape $ABCD$.

G8-M2-Lesson 8: Sequencing Reflections and Translations

1. Let there be a reflection across line L, and let there be a translation along vector $\overrightarrow{HJ}$. Compare the translation of Figure S followed by the reflection of Figure S with the reflection of Figure S followed by the translation of Figure S. What do you notice?

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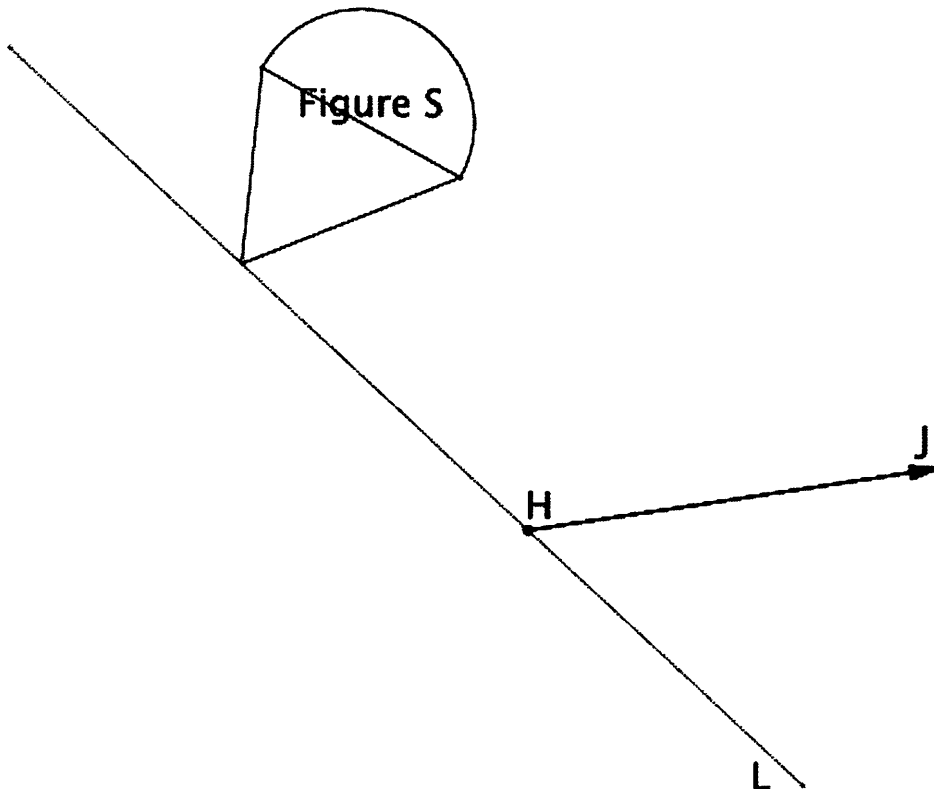
Sample student response.
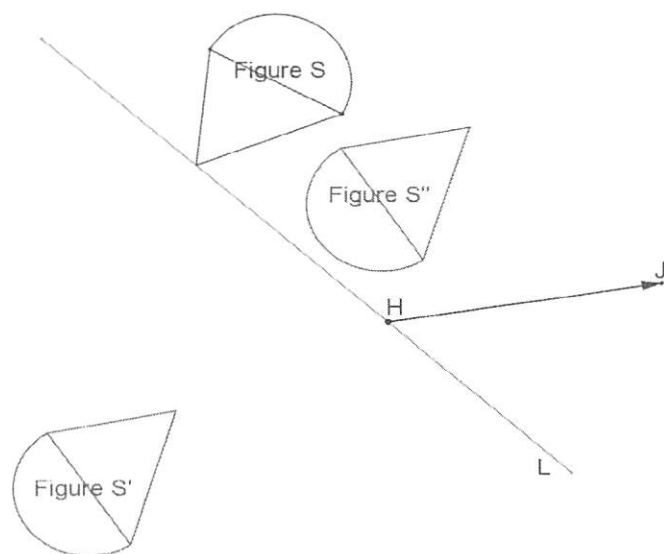
I remember my teacher saying the order in which the rigid motions are performed matters. Translation of Figure S followed by the reflection of Figure S maps to Figure S'.

Reflection of Figure S followed by the translation of Figure S maps to Figure S''.

Students should notice that the two sequences place Figure S in different locations in the plane.

2. Let L_1 and L_2 be parallel lines, and let $Reflection_1$ and $Reflection_2$ be the reflections across L_1 and L_2, respectively (in that order). Can you guess what $Reflection_1$ followed by $Reflection_2$ is? Give as persuasive an argument as you can.

The sequence $Reflection_1$ followed by $Reflection_2$ is just like the translation along a vector $\overrightarrow{EF}$, as shown below, where $\overrightarrow{EF}$ is perpendicular to L_1. The length of $\overrightarrow{EF}$ is equal to twice the distance between L_1 and L_2.
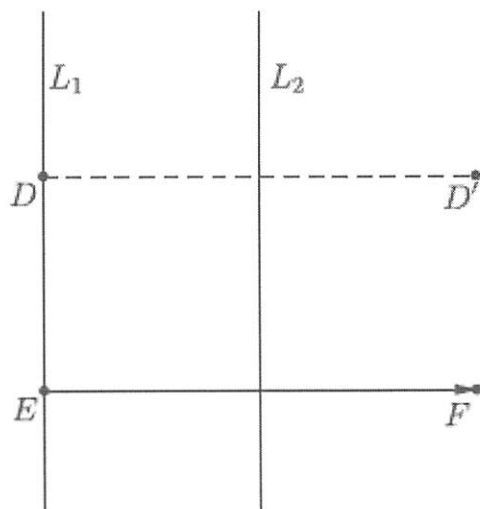

I'm going to draw a diagram to help me explain. I will pick a point D on L_1 and reflect it across line L_1 first, and then I'll reflect across line L_2. When I did the first reflection, point D stayed on L_1 since it is on the line of reflection. When I did the second reflection, I noticed that the point D' looked like it had been translated along a vector. I checked with my transparency.

G8-M2-Lesson 9: Sequencing Rotations

Refer to the figure below to answer Problems 1–3. Note: Figure is not drawn to scale.
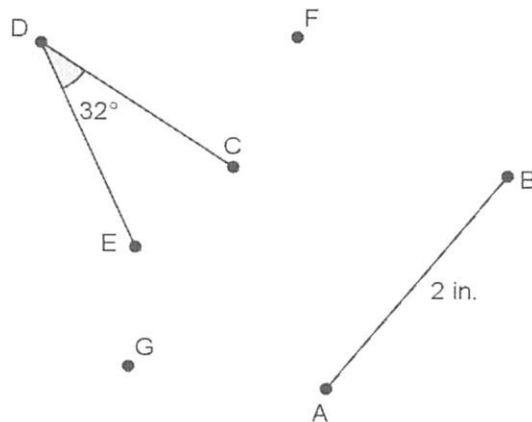
1. Rotate ∠CDE and segment AB d degrees around center F and then
   d degrees around center G. Label the final location of the images as
   ∠C′D′E′ and segment A′B′.

   Drawings will vary based on students' choice of degree to rotate.
   Shown below is a rotation around point F of 45° followed by a rotation
   around point G of 80°.

   > The order in
   > which I rotate
   > matters because
   > I am using two
   > different centers.

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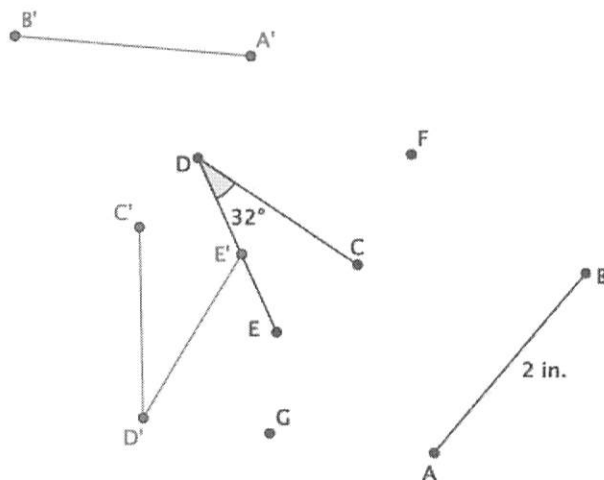
2. What is the measure of $\angle CDE$, and how does it compare to the measure of $\angle C'D'E'$? Explain.

    The measure of $\angle CDE$ is $32°$. The measure of $\angle C'D'E'$ is $32°$. The angles are equal in measure because a sequence of rotations preserves the degrees of an angle.

3. What is the length of segment AB, and how does it compare to the length of segment $A'B'$? Explain.

    The length of segment AB is 2 in. The length of segment $A'B'$ is also 2 in. The segments are equal in length because a sequence of rotations preserves the length of a segment.

Refer to the figure below to answer Problem 4.
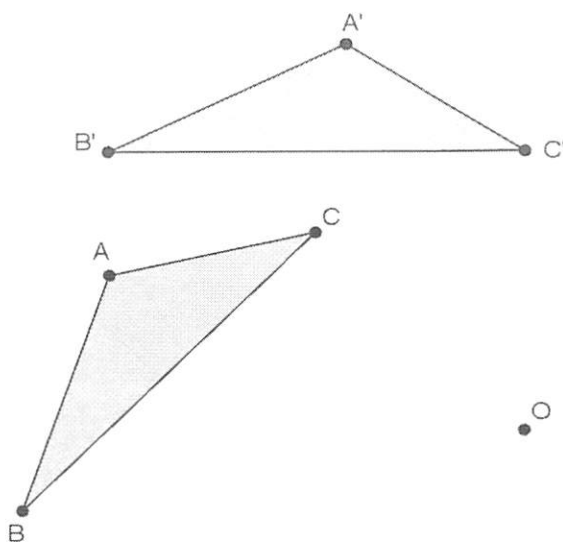
When I rotate in the positive direction, I rotate counterclockwise. When I rotate in the negative direction, I rotate clockwise.

4. Let $Rotation_1$ be a rotation of $45°$ around the center O. Let $Rotation_2$ be a rotation of $-90°$ around the same center O. Determine the approximate location of $Rotation_1(\triangle ABC)$ followed by $Rotation_2(\triangle ABC)$. Label the image of $\triangle ABC$ as $\triangle A'B'C'$.

    The image of $\triangle ABC$ is shown above and labeled $\triangle A'B'C'$.

G8-M2-Lesson 10: Sequences of Rigid Motions

1. Let there be a reflection across the y-axis, let there be a translation along vector $\vec{u}$, and let there be a rotation around point A, 90° (counterclockwise). Let S be the figure as shown below. Show the location of S after performing the following sequence: a reflection followed by a translation followed by a rotation. Label the image as Figure S'.
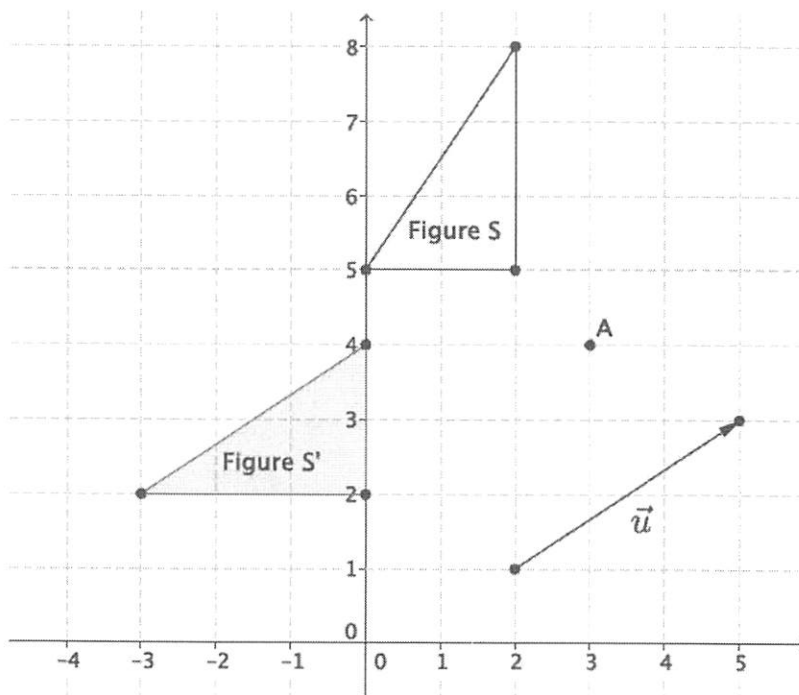
> I remember my teacher saying the order matters. I can use my transparency to perform the sequence.

2. Would the location of the image of S in the previous problem be the same if the translation was performed last instead of second; that is, does the sequence, reflection followed by a rotation followed by a translation, equal a reflection followed by a translation followed by a rotation? Explain.

No, the order of the transformations matters. If the translation was performed last, the location of the image of S, after the sequence, would be in a different location than if the translation was performed second.

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G8-M2-Lesson 11: Definition of Congruence and Some Basic Properties

Are the two parallelograms shown below congruent? If so, describe a congruence that would map one parallelogram onto the other.

> To prove two figures are congruent, I have to show that one figure will map onto another using a sequence of rigid motions. I could try it first with my transparency.

> I remember my teacher saying it makes more sense to translate the figure along a vector first to get a common point and then rotate the figure about the point to get a common side that I can then use as the line of reflection.
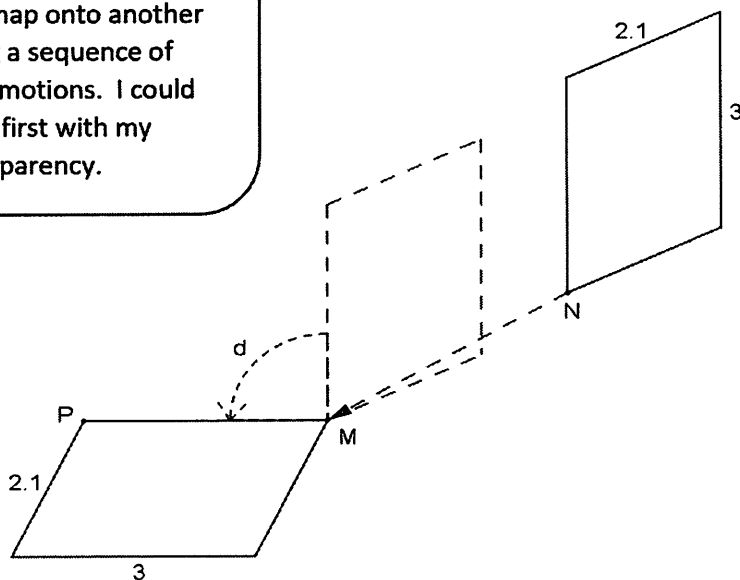
Sample student response: Yes, they are congruent. Let there be a translation along vector $\overrightarrow{NM}$. Let there be a rotation around point M, d degrees. Let there be a reflection across line MP. Then, the translation followed by the rotation followed by the reflection will map the parallelogram on the right to the parallelogram on the left.

> The diagram doesn't have any points noted. If I'm going to be precise, I'll have to add the points N, M, and P to the drawing.

G8-M2-Lesson 12: Angles Associated with Parallel Lines

Use the diagram below to complete Problems 1–2.
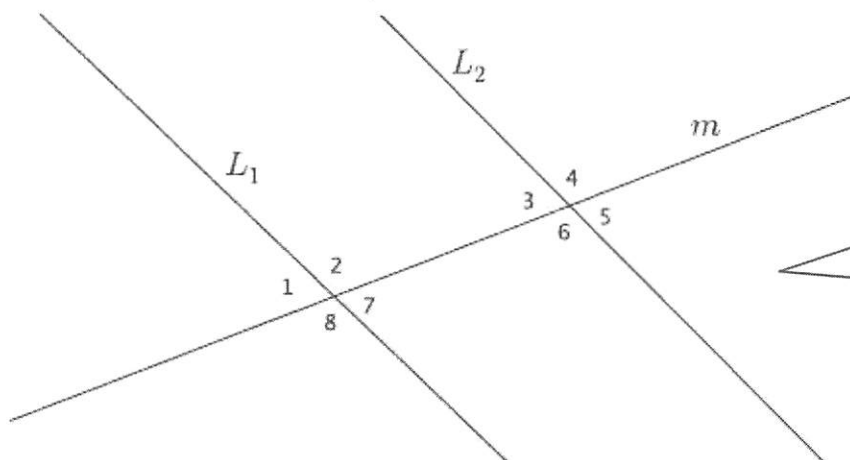
Even though lines L_1 and L_2 might look parallel, I can't assume that they are. Since line m crosses lines L_1 and L_2, then line m is the transversal.

1. Identify all pairs of corresponding angles. Are the pairs of corresponding angles equal in measure? How do you know?

    ∠1 and ∠3, ∠2 and ∠4, ∠8 and ∠6, ∠7 and ∠5

    Corresponding angles are on the same side of the transversal in corresponding positions.

    There is no information provided about the lines in the diagram being parallel. For that reason, we do not know if the pairs of corresponding angles are equal in measure. If we knew the lines were parallel, we could use a translation to map one angle onto another.
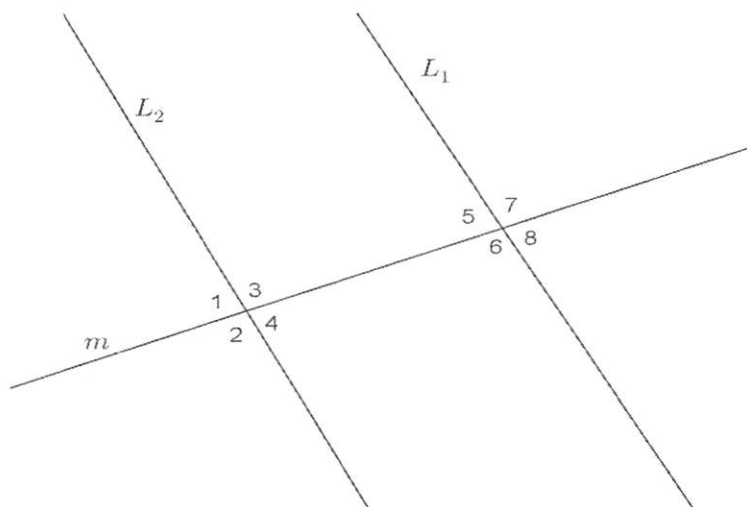
2. Identify all pairs of alternate interior angles. Are the pairs of alternate interior angles equal in measure? How do you know?

    ∠2 and ∠6, ∠3 and ∠7

    Alternate interior angles are on opposite sides of the transversal in between L_1 and L_2.

    There is no information provided about the lines in the diagram being parallel. For that reason, we do not know if the pairs of alternate interior angles are equal in measure. If the lines were parallel, we could use a rotation to show that the pairs of angles would map onto one another, proving they are equal in measure.

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Use the diagram below to complete Problems 3–4. In the diagram, $L_1 \parallel L_2$.

> When parallel lines are cut by a transversal, pairs of corresponding angles, alternate interior angles, and alternate exterior angles are equal.

3. Use an informal argument to describe why ∠1 and ∠8 are equal in measure.

The reason that ∠1 and ∠8 are equal in measure when the lines are parallel is because you can rotate around the midpoint of the segment between the parallel lines. A rotation would then map ∠1 onto ∠8, showing that they are congruent and equal in measure.

> I remember my teacher marking the midpoint between L_1 and L_2 on line m and demonstrating the rotation. I remember from Lesson 5 that rotation preserves degrees of angles.

4. Use an informal argument to describe why ∠1 and ∠5 are equal in measure.

The reason that ∠1 and ∠5 are equal in measure when the lines are parallel is because you can translate along a vector equal in length of the segment between the parallel lines; then, ∠1 would map onto ∠5.

> I remember from Lesson 2 that translations along a vector preserve degrees of angles. My teacher demonstrated this in class by translating along a vector on line m the exact distance between L_1 and L_2.

G8-M2-Lesson 13: Angle Sum of a Triangle

1. In the diagram below, line AB is parallel to line EF; that is, $L_{AB} \parallel L_{EF}$. The measure of $\angle BAH$ is 21°, and the measure of $\angle FEH$ is 36°. Find the measure of $\angle AHE$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Extend the segment EH so that it intersects line AB.)

E
36°

A
21°   H   F

G

B

I extended segment EH so that it intersects line AB. I labeled the intersection point G. This creates straight angle $\angle EHG$, and the measure of a straight angle is 180°. This also creates triangle AGH, and the sum of the interior angles of a triangle is always 180°.
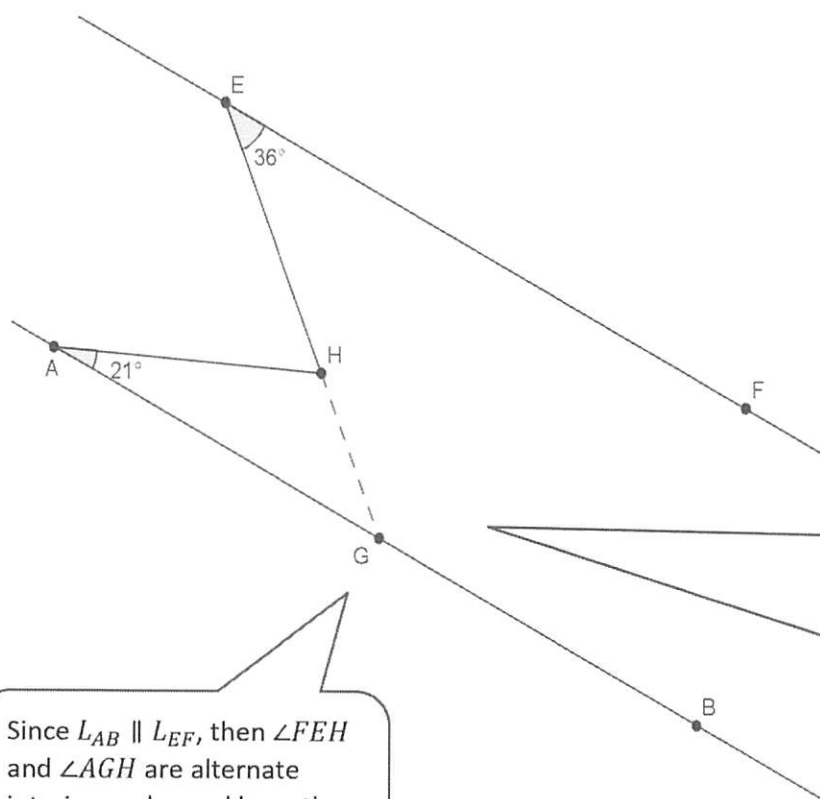
Since $L_{AB} \parallel L_{EF}$, then $\angle FEH$ and $\angle AGH$ are alternate interior angles and have the same measure.

Since $\angle FEH$ and $\angle AGH$ are alternate interior angles of parallel lines, the angles are congruent and have the same measure. Since the angle sum of a triangle is 180°, the measure of $\angle AHG$ is $180° - (36° + 21°)$ which is equal to 123°. The straight angle $\angle EHG$ is made up of $\angle AHG$ and $\angle EHA$. Since straight angles measure 180° and the measure of $\angle AHG$ is 123°, then the measure of $\angle AHE$ is 57°.

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2.   What is the measure of $\angle CAB$?
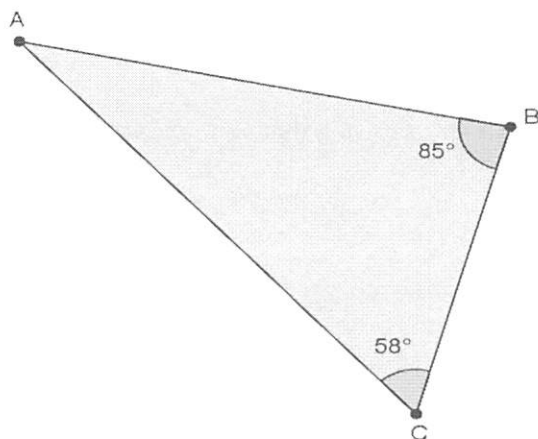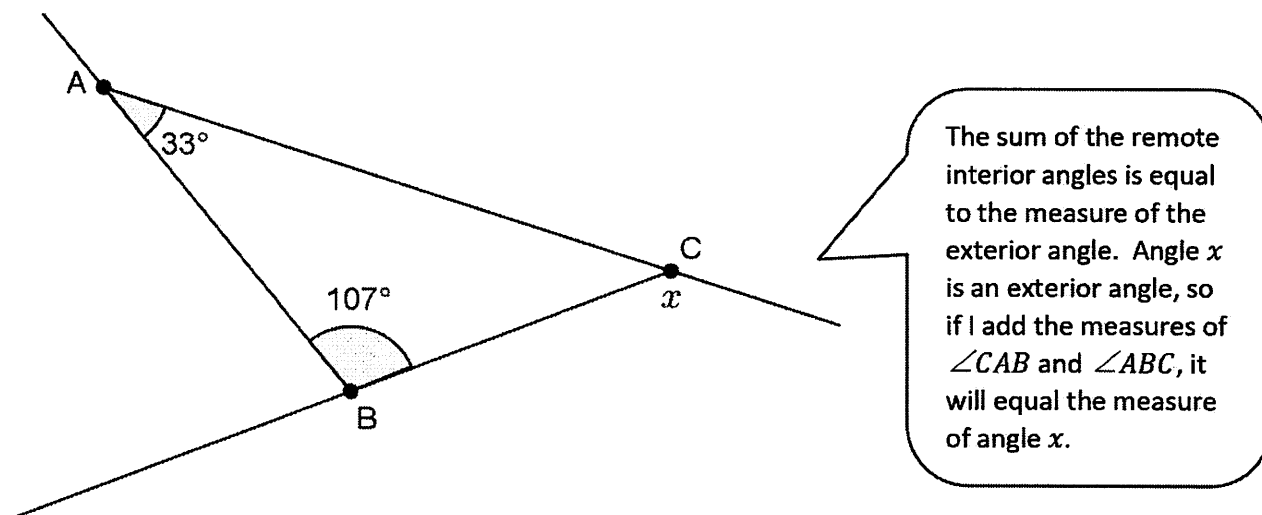

I know if I add up all the interior angles of a triangle, they will equal 180°.

The measure of $\angle CAB$ is $180° - (85° + 58°)$, which is equal to $37°$.

G8-M2-Lesson 14: More on the Angles of a Triangle

1.   Find the measure of angle x. Present an informal argument showing that your answer is correct.

A
33°

107°

B

C
x

> The sum of the remote interior angles is equal to the measure of the exterior angle. Angle x is an exterior angle, so if I add the measures of $\angle CAB$ and $\angle ABC$, it will equal the measure of angle x.

Since $33 + 107 = 140$, the measure of angle x is $140°$. We know that triangles have a sum of interior angles that is equal to $180°$. We also know that straight angles are $180°$. The measure of $\angle ACB$ must be $40°$, which means that the measure of $\angle x$ is $140°$.

> There is a straight angle comprised of $\angle ACB$ and $\angle x$ that will equal $180°$.

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2. Write an equation that would allow you to find the measure of $\angle y$. Present an informal argument showing that your answer is correct.
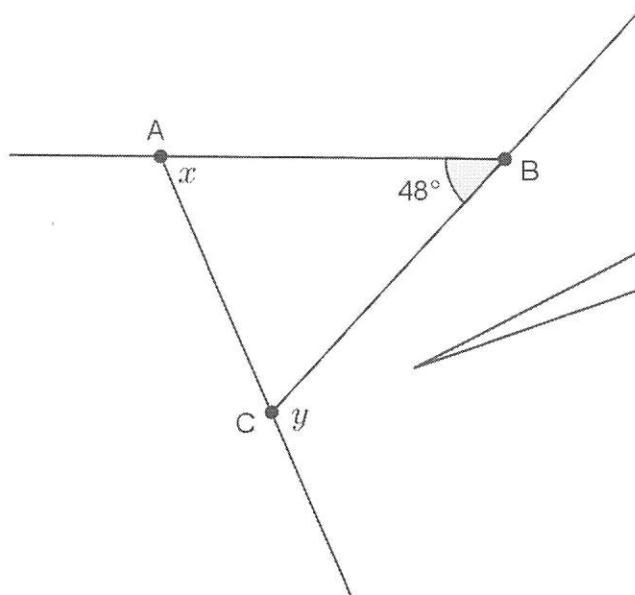
> I know the sum of the remote interior angles, $48° + x$, is equal to the exterior angle, y.

> The sum of interior angles of a triangle is $180°$, and the straight angle is $180°$. I can write an equation using these two facts.

Since $48° + x = y$, the measure of $\angle y$ is $48° + x$.

We know that triangles have a sum of interior angles that is equal to $180°$. We also know that straight angles are $180°$.

> The measure of the interior angles is $x + 48° + \angle ACB$.

> The measure of the straight angle is $y + \angle ACB$.

Then, $x + 48° + \angle ACB = 180°$, and $y + \angle ACB = 180°$. Since both equations are equal to $180°$, then $x + 48° + \angle ACB = y + \angle ACB$. Subtracting $\angle ACB$ from each side of the equation yields $x + 48° = y$.

G8-M2-Lesson 15: Informal Proof of the Pythagorean Theorem

For each of the problems below, determine the length of the hypotenuse of the right triangle shown. Note: Figures are not drawn to scale.
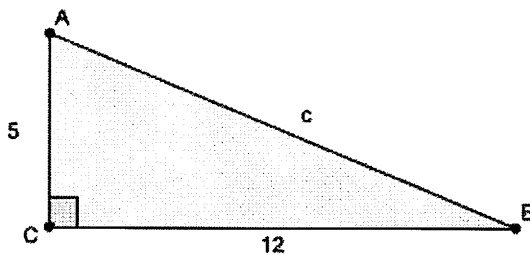
1.

$$a^2 + b^2 = c^2$$
$$2^2 + 4^2 = c^2$$
$$4 + 16 = c^2$$
$$20 = c^2$$
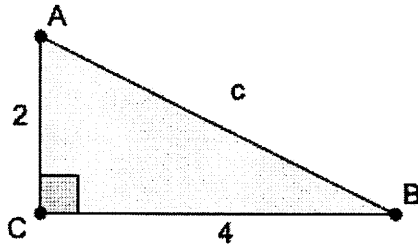
I know that 2 and 4 are the legs of the triangle. I know this because the hypotenuse is across from the 90° angle. Since the hypotenuse is side c in my formula, I substitute the 2 and 4 for a and b.

Since I do not know what number times itself produces 20, for now I can leave my answer as $20 = c^2$.

2.

$$a^2 + b^2 = c^2$$
$$12^2 + 5^2 = c^2$$
$$144 + 25 = c^2$$
$$169 = c^2$$
$$13 = c$$

Since I know that $13 \times 13 = 169$, then I know that $c = 13$.

EUREKA MATH

G8-M2-Lesson 16: Applications of the Pythagorean Theorem

1. Find the length of the segment AB shown below, if possible.

> I know that the grid lines on the coordinate plane meet at a right angle. I can make my right triangle using a horizontal line through point A and a vertical line through point B.
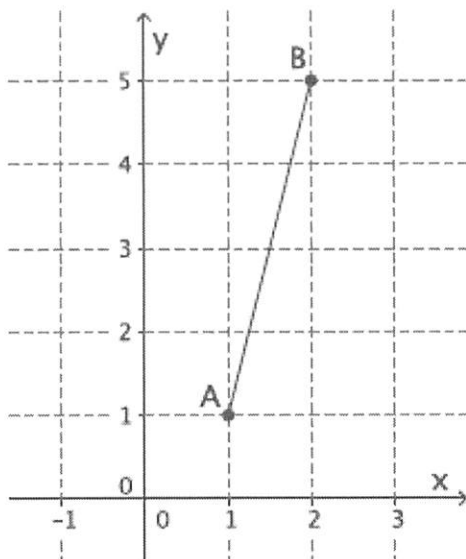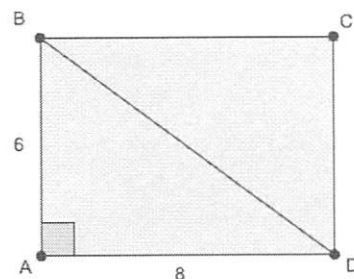
$$a^2 + b^2 = c^2$$
$$1^2 + 4^2 = c^2$$
$$1 + 16 = c^2$$
$$17 = c^2$$
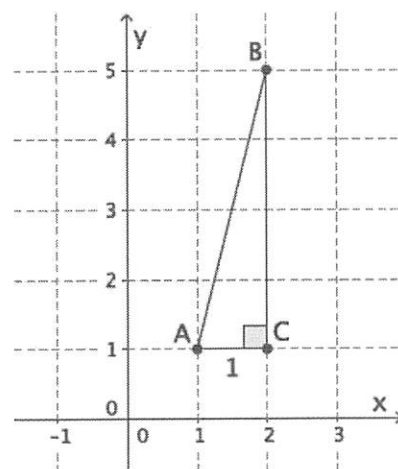
2. A rectangle has dimensions 6 cm by 8 cm. What is the length of the diagonal of the rectangle?

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = c^2$$
$$36 + 64 = c^2$$
$$100 = c^2$$
$$10 = c$$

The length of the diagonal is 10 cm.

> I should draw the rectangle so I can see the right triangle.

3. Determine the length of the unknown side, if possible.
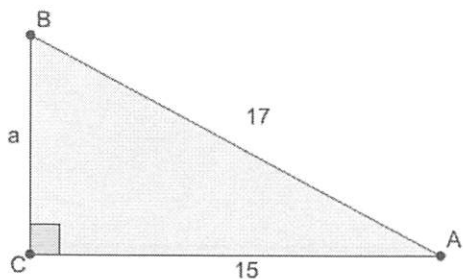
I know that the hypotenuse is 17 and that the hypotenuse is represented by c in my formula. This time, I need to substitute for b and c and then solve the equation to find the length of the missing leg.

$$a^2 + b^2 = c^2$$
$$a^2 + 15^2 = 17^2$$
$$a^2 + 225 = 289$$
$$a^2 + 225 - 225 = 289 - 225$$
$$a^2 = 64$$
$$a = 8$$

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