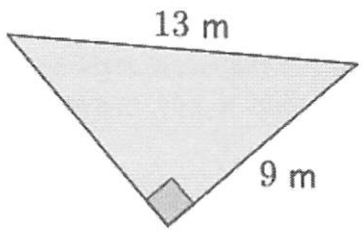
Homework Helpers

Grade 8
Module 7

G8-M7-Lesson 1: The Pythagorean Theorem

I remember the Pythagorean theorem from previous lessons.

1. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.



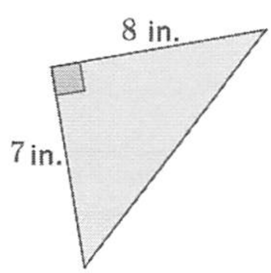
Let x m represent the length of the unknown side.

$$9^2 + x^2 = 13^2$$
$$81 + x^2 = 169$$
$$x^2 = 88$$

The number 88 is not a perfect square, but I do know that 81 and 100 are perfect squares, and 88 is between them.

The number 88 is between the perfect squares 81 and 100. Since 88 is closer to 81 than it is to 100, the length of the unknown side of the triangle is closer to 9 m than it is to 10 m.

2. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.

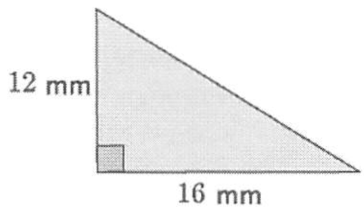


Let c in. represent the length of the hypotenuse.

$$7^2 + 8^2 = c^2$$
$$49 + 64 = c^2$$
$$113 = c^2$$

The number 113 is between the perfect squares 100 and 121. Since 113 is closer to 121 than it is to 100, the length of the hypotenuse of the triangle is closer to 11 in. than it is to 10 in.

3. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.



Let c mm represent the length of the hypotenuse.

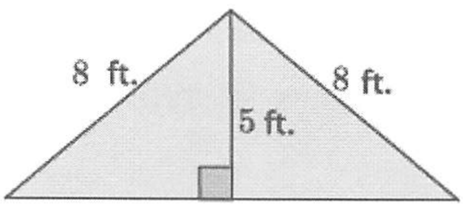
$$12^2 + 16^2 = c^2$$
$$144 + 256 = c^2$$
$$400 = c^2$$
$$20 = c$$

The number 400 is a perfect square, so I know that 20 is equal to c.

The length of the hypotenuse is 20 mm. The Pythagorean theorem led me to the fact that the square of the unknown side is 400. We know 400 is a perfect square, and 400 is equal to 20^2; therefore, $c = 20$, and the length of the hypotenuse of the triangle is 20 mm.

4. The triangle below is an isosceles triangle. Use what you know about the Pythagorean theorem to determine the approximate length of the base of the isosceles triangle.

I can find the length of the base of one triangle and multiply that number by 2 since the two right triangles are congruent.

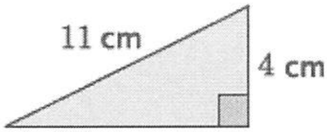


Let x ft. represent the length of the base of one of the right triangles of the isosceles triangle.

$$x^2 + 5^2 = 8^2$$
$$x^2 + 25 = 64$$
$$x^2 = 39$$

Since 39 is between the perfect squares 36 and 49 but closer to 36, the approximate length of the base of the right triangle is 6 ft. Since there are two right triangles, the length of the base of the isosceles triangle is approximately 12 ft.

5. Give an estimate for the area of the triangle shown below. Explain why it is a good estimate.



Let x cm represent the length of the base of the right triangle.

$$x^2 + 4^2 = 11^2$$
$$x^2 + 16 = 121$$
$$x^2 = 105$$

I need to use the base, b, and height, h, of the triangle to find the area, A, of a triangle; $A = \frac{1}{2}bh$.

Since 105 is between the perfect squares 100 and 121 but closer to 100, the approximate length of the base is 10 cm. $A = \frac{1}{2}(10)(4) = 20$. So, the approximate area of the triangle is 20 cm^2.

The hypotenuse is the longest side, so the base must be less than 11 cm.

20 cm^2 is a good estimate because of the approximation of the length of the base. Furthermore, since the hypotenuse is the longest side of the right triangle, approximating the length of the base as 10 cm makes mathematical sense because it has to be shorter than the hypotenuse.

G8-M7-Lesson 2: Square Roots

Determine the positive square root of the number given. If the number is not a perfect square, determine the integer to which the square root would be closest.

1. $\sqrt{196}$

   14

   I know that perfect squares have square roots that are equal to integers.

2. $\sqrt{225}$

   15

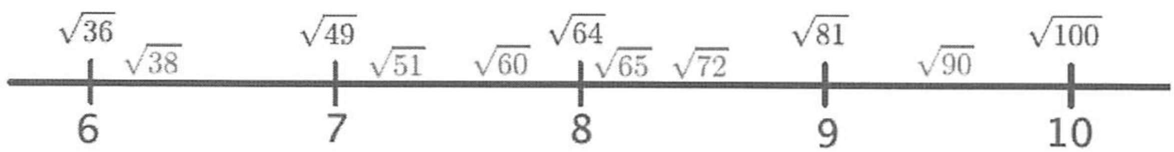
3. Between which two integers will $\sqrt{22}$ be located? Explain how you know.

   The number 22 is not a perfect square. It is between the perfect squares 16 and 25 but closer to 25. Therefore, the square root of 25 is between the integers 4 and 5 because $\sqrt{16} = 4$ and $\sqrt{25} = 5$ and $\sqrt{16} < \sqrt{22} < \sqrt{25}$.

4. Place the following list of numbers in their approximate locations on a number line.

   $\sqrt{60}$, $\sqrt{38}$, $\sqrt{65}$, $\sqrt{90}$, $\sqrt{72}$, and $\sqrt{51}$

   I can use the same reasoning from Problem 2 to determine the placement of the square roots on the number line.



Answers are noted in blue.

G8-M7-Lesson 3: Existence and Uniqueness of Square Roots and Cube Roots

1. A square-shaped park has an area of 900 ft^2. What are the dimensions of the park? Write and solve an equation.

Let x ft. represent the length of one side of the park.

$$x^2 = 900$$
$$\sqrt{x^2} = \sqrt{900}$$
$$x = \sqrt{900}$$
$$x = 30$$

Check:

$$30^2 = 900$$
$$900 = 900$$

The square park is 30 ft. in length and 30 ft. in width.

2. A cube has a volume of 125 in^3. What is the measure of one of its sides? Write and solve an equation.

Let x in. represent the length of one side of the cube.

I need to use the cube root symbol, namely $\sqrt[3]{x}$, to find the cube root of a number x.

$$x^3 = 125$$
$$\sqrt[3]{x^3} = \sqrt[3]{125}$$
$$x = \sqrt[3]{125}$$
$$x = 5$$

Check:

$$5^3 = 125$$
$$125 = 125$$

The cube has a side length of 5 in.

3. Find the value of x that makes the equation true: $x^3 = 216^{-1}$.

$$x^3 = 216^{-1}$$
$$\sqrt[3]{x^3} = \sqrt[3]{216^{-1}}$$
$$x = \sqrt[3]{216^{-1}}$$
$$x = \sqrt[3]{\frac{1}{216}}$$
$$x = \frac{1}{6}$$
$$x = 6^{-1}$$

I remember that I can rewrite 216^{-1} as $\frac{1}{216}$ to make the problem easier.

Check:

$$\left(6^{-1}\right)^3 = 216^{-1}$$
$$6^{-3} = 216^{-1}$$
$$\frac{1}{6^3} = 216^{-1}$$
$$\frac{1}{216} = 216^{-1}$$
$$216^{-1} = 216^{-1}$$

4. Find the positive value of x that makes the following equation true: $x^2 - 22 = 99$.

$$x^2 - 22 = 99$$
$$x^2 - 22 + 22 = 99 + 22$$
$$x^2 = 121$$
$$\sqrt{x^2} = \sqrt{121}$$
$$x = 11$$

The positive value for x that makes the equation true is 11.

G8-M7-Lesson 4: Simplifying Square Roots

Simplify each of the square roots in Problems 1–4 as much as possible.

1. $\sqrt{8}$

$$\begin{aligned}\sqrt{8} &= \sqrt{2\times 4}\\ &= \sqrt{2\times 2^2}\\ &= 2\times\sqrt{2}\\ &= 2\sqrt{2}\end{aligned}$$

I need to rewrite 8 with factors of perfect squares.

2. $\sqrt{150}$

I can rewrite $\sqrt{2}\times\sqrt{3}=\sqrt{2\times 3}=\sqrt{6}$.

$$\begin{aligned}\sqrt{150} &= \sqrt{2\times 3\times 5^2}\\ &= \sqrt{2}\times\sqrt{3}\times\sqrt{5^2}\\ &= 5\times\sqrt{2\times 3}\\ &= 5\sqrt{6}\end{aligned}$$

3. $\sqrt{3675}$

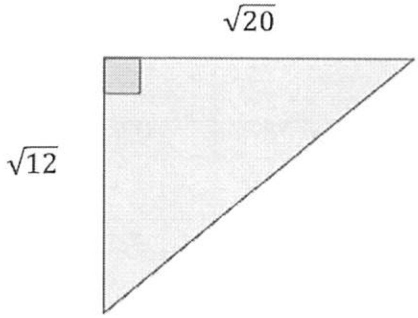
$$\begin{aligned}\sqrt{3675} &= \sqrt{3\times 5^2\times 7^2}\\ &= \sqrt{3}\times\sqrt{5^2}\times\sqrt{7^2}\\ &= 5\times 7\times\sqrt{3}\\ &= 35\sqrt{3}\end{aligned}$$

Since $\sqrt{3}$ is not a perfect square, I will leave it as it is.

4. $\sqrt{121}$

$$\begin{aligned}\sqrt{121} &= \sqrt{11^2}\\ &= 11\end{aligned}$$

5. What is the length of the unknown side of the right triangle? Simplify your answer, if possible.



Let c units represent the length of the hypotenuse.

$$\left(\sqrt{20}\right)^2 + \left(\sqrt{12}\right)^2 = c^2$$
$$20 + 12 = c^2$$
$$32 = c^2$$
$$\sqrt{32} = \sqrt{c^2}$$
$$\sqrt{4^2} \times \sqrt{2} = c$$
$$4\sqrt{2} = c$$

Using my laws of exponents, I know that $\left(\sqrt{20}\right)^2$ means $\sqrt{20} \times \sqrt{20}$, which means $\sqrt{20 \times 20} = \sqrt{20^2} = 20$.

The length of the unknown side of the right triangle is $4\sqrt{2}$ units.

G8-M7-Lesson 5: Solving Equations with Radicals

1. Find the positive value of x that makes each equation true, and then verify that your solution is correct.

$$x^2 + 6x - 12 = 6(x + 4)$$

Using my properties of equalities and the distributive property, I want to rewrite the equation in the form of $x^2 = p$, where p is a positive rational number, and then take the square root and determine the value of x.

$$\begin{aligned} x^2 + 6x - 12 &= 6(x + 4) \\ x^2 + 6x - 12 &= 6x + 24 \\ x^2 + 6x - 12 + 12 &= 6x + 24 + 12 \\ x^2 + 6x &= 6x + 36 \\ x^2 + 6x - 6x &= 6x - 6x + 36 \\ x^2 &= 36 \\ x &= 6 \end{aligned}$$

Check:

$$\begin{aligned} 6^2 + 6(6) - 12 &= 6(6 + 4) \\ 36 + 36 - 12 &= 6(10) \\ 60 &= 60 \end{aligned}$$

2. Determine the positive value of x that makes the equation true, and then explain how you solved the equation.

$$\frac{x^{11}}{x^8} - 125 = 0$$

I can rewrite the equation using the laws of exponents.

$$\begin{aligned} \frac{x^{11}}{x^8} - 125 &= 0 \\ x^3 - 125 &= 0 \\ x^3 - 125 + 125 &= 0 + 125 \\ x^3 &= 125 \\ \sqrt[3]{x^3} &= \sqrt[3]{125} \\ x &= \sqrt[3]{5^3} \\ x &= 5 \end{aligned}$$

Check:

$$\begin{aligned} \frac{5^{11}}{5^8} - 125 &= 0 \\ 5^3 - 125 &= 0 \\ 125 - 125 &= 0 \\ 0 &= 0 \end{aligned}$$

To solve the equation, I first had to simplify the expression $\frac{x^{11}}{x^8}$ to x^3. Next, I used the properties of equality to transform the equation into $x^3 = 125$. Finally, I had to take the cube root of both sides of the equation to solve for x.

3. Determine the positive value of x that makes the equation true.

$$\left(5\sqrt{x}\right)^2 - 7x = 72$$

I only need to find the positive value of x that makes the equation true.

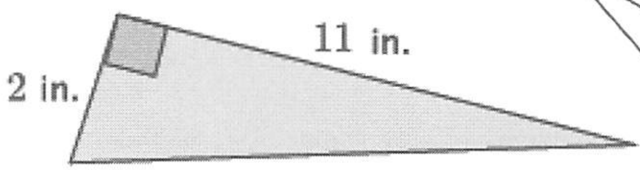
$$\begin{aligned}\left(5\sqrt{x}\right)^2 - 7x &= 72\\ 5^2(\sqrt{x})^2 - 7x &= 72\\ 25x - 7x &= 72\\ 18x &= 72\\ \frac{18x}{18} &= \frac{72}{18}\\ x &= 4\end{aligned}$$

Check:

$$\begin{aligned}\left(5\sqrt{4}\right)^2 - 7(4) &= 72\\ 5^2\left(\sqrt{4}\right)^2 - 28 &= 72\\ 25(4) - 28 &= 72\\ 100 - 28 &= 72\\ 72 &= 72\end{aligned}$$

4. Determine the length of the hypotenuse of the right triangle below.

The lengths of the sides of a triangle are positive, so a negative answer would not make sense.



I must define my variable.

Let x in. represent the length of the hypotenuse.

$$\begin{aligned}2^2 + 11^2 &= x^2\\ 4 + 121 &= x^2\\ 125 &= x^2\\ \sqrt{125} &= \sqrt{x^2}\\ \sqrt{5^2 \cdot 5} &= x\\ 5\sqrt{5} &= x\end{aligned}$$

Check:

I am using the laws of exponents to check my answer.

$$\begin{aligned}2^2 + 11^2 &= \left(5\sqrt{5}\right)^2\\ 4 + 121 &= 5^2\left(\sqrt{5}\right)^2\\ 125 &= 25(5)\\ 125 &= 125\end{aligned}$$

Since $x = 5\sqrt{5}$, then the length of the hypotenuse is $5\sqrt{5}$ in.

This is an exact answer and cannot be simplified any further.

G8-M7-Lesson 6: Finite and Infinite Decimals

Convert each fraction to a finite decimal. If the fraction cannot be written as a finite decimal, then state how you know. Show your steps, but use a calculator for the multiplication.

1. $\frac{13}{16}$

The denominator 16 *is equal to* 2^4.

$$\frac{13}{16}=\frac{13}{2^4}=\frac{13\times 5^4}{2^4\times 5^4}=\frac{13\times 5^4}{(2\times 5)^4}=\frac{8125}{10^4}=0.8125$$

The denominator 16 is equal to 2^4. I need to multiply the denominator by 4 factors of 5 so that the denominator is a multiple of 10, and I can write the fraction easily.

2. $\frac{2}{125}$

The denominator 125 *is equal to* 5^3.

$$\frac{2}{125}=\frac{2}{5^3}=\frac{2\times 2^3}{2^3\times 5^3}=\frac{2^4}{(2\times 5)^3}=\frac{16}{10^3}=0.016$$

I need to multiply the denominator by 3 factors of 2.

3. $\frac{22}{75}$

The fraction $\frac{22}{75}$ *is not a finite decimal because the denominator* 75 *is equal to* 3×5^2. *The denominator cannot be expressed as a product of* 2*'s and* 5*'s; therefore,* $\frac{22}{75}$ *is not a finite decimal.*

The denominator cannot be written as a product of 2's and/or 5's.

4. $\frac{33}{800}$

The denominator 800 *is equal to* $2^5\times 5^2$.

$$\frac{33}{800}=\frac{33}{2^5\times 5^2}=\frac{33\times 5^3}{2^5\times 5^2\times 5^3}=\frac{33\times 5^3}{2^5\times 5^5}=\frac{33\times 5^3}{(2\times 5)^5}=\frac{4125}{10^5}=0.04125$$

To make the powers of the bases match, I need to multiply 5^2 by 5^3.

G8-M7-Lesson 7: Infinite Decimals

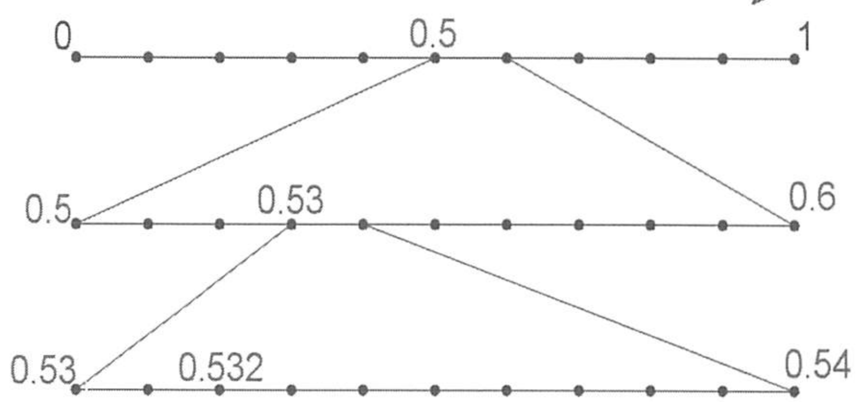
1.

a. Write the expanded form of the decimal 0.532 using powers of 10.

$$0.532 = \frac{5}{10} + \frac{3}{10^2} + \frac{2}{10^3}$$

The first number line is divided into 10 equal parts, namely tenths. The next number line is divided into 10 equal parts, namely hundredths. The third number line is divided into 10 equal parts, namely thousandths.

b. Show on the number line the representation of the decimal 0.532.



c. Is the decimal finite or infinite? How do you know?

The decimal 0.532 is finite because it can be completely represented by a finite number of steps in the sequence.

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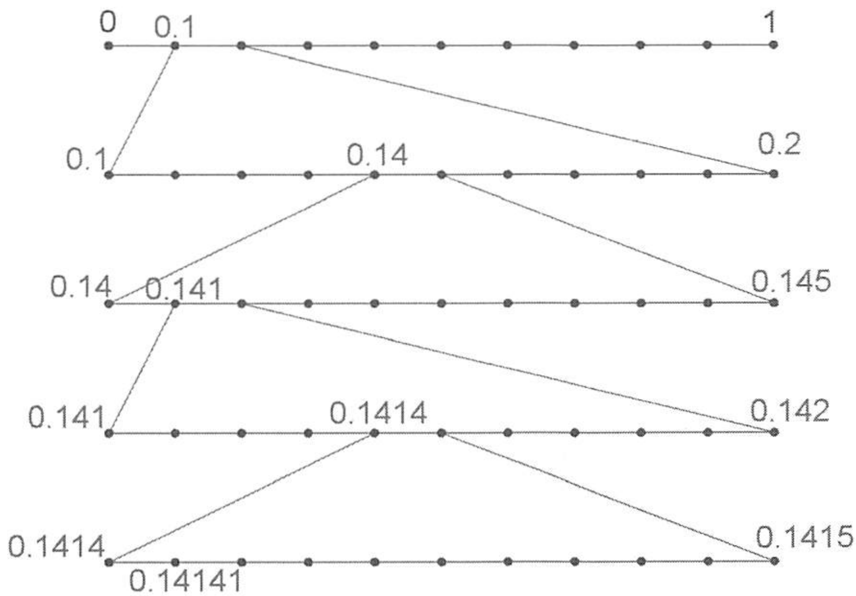
2.

a. Write the expanded form of the decimal $0.\overline{14}$ using powers of 10.

$$0.\overline{14} = \frac{1}{10} + \frac{4}{10^2} + \frac{1}{10^3} + \frac{4}{10^4} + \frac{1}{10^5} + \frac{4}{10^6} + \cdots$$

b. Show on the number line the representation of the decimal $0.141414....$



I see that the number $0.\overline{14}$ is represented by an infinite number of steps. I can keep drawing number lines, but I will never find the precise location of $0.\overline{14}$.

c. Is the decimal finite or infinite? How do you know?

The decimal $0.\overline{14}$ is infinite because it cannot be represented by a finite number of steps. Because the digits 1 and 4 continue to repeat, there will be an infinite number of steps in the sequence.

3. Explain why $0.444 < 0.4444$.

Rewriting each decimal in its expanded form will help me to explain which is larger.

The number $0.444 = \frac{4}{10} + \frac{4}{10^2} + \frac{4}{10^3}$, and the number $0.4444 = \frac{4}{10} + \frac{4}{10^2} + \frac{4}{10^3} + \frac{4}{10^4}$. That means that 0.4444 is exactly $\frac{4}{10^4}$ larger than 0.444. If we examine the numbers on the number line, 0.4444 is to the right of 0.444, meaning that it is larger than 0.444.

G8-M7-Lesson 8: The Long Division Algorithm

1. Write the decimal expansion of $\frac{4000}{3}$. Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

$$\frac{4000}{3} = \frac{1333 \times 3}{3} + \frac{1}{3}$$
$$= 1333\frac{1}{3}$$

I know that the decimal expansion will be infinite since the divisor is not a product of 2's and 5's. I need to use the long division algorithm.

$$
\begin{array}{r}
1\,3\,3\,3\,.\,3\,3 \\
3\,\overline{)\,4\,0\,0\,0\,.\,0\,0} \\
-\underline{3} \\
1\,0 \\
-\underline{9} \\
1\,0 \\
-\underline{9} \\
1\,0 \\
-\underline{9} \\
1\,0 \\
-\underline{9} \\
1\,0 \\
-\underline{9} \\
1\,0
\end{array}
$$

The remainder is repeating, so I can stop dividing.

The decimal expansion of $\frac{4000}{3}$ *is* $1333.\overline{3}$. *The number is rational because it has the repeating digit of* 3. *Rational numbers have decimal expansions that repeat; therefore,* $\frac{4000}{3}$ *is a rational number.*

2. Write the decimal expansion of $\frac{3{,}888{,}885}{11}$. Based on our definition of rational numbers having a decimal expansion that repeats eventually, is the number rational? Explain.

$$\frac{3\,888\,885}{11} = \frac{353\,535 \times 11}{11} + \frac{0}{11}$$
$$= 353\,535$$

The $+\frac{0}{11}$ means that the remainder is a repeating block of zeros.

The decimal expansion of $\frac{3\,885\,885}{11}$ *is* $353{,}535$. *The number is rational because we can write the repeating digit of* 0 *following the whole number. Rational numbers have decimal expansions that repeat; therefore,* $\frac{3\,888\,885}{11}$ *is a rational number.*

3. Someone notices that the long division of 1,855,855 by 9 has a quotient of 206,206 and a remainder of 1 and wonders why there is a repeating block of digits in the quotient, namely 206. Explain to the person why this happens.

$$\begin{array}{r|rrrrrrr}
 & & 2 & 0 & 6 & 2 & 0 & 6 \\ \cline{2-8}
9 & 1 & 8 & 5 & 5 & 8 & 5 & 5 \\
- & 1 & 8 & & & & & \\ \cline{2-8}
 & & 0 & 5 & & & & \\
 & - & & 0 & & & & \\ \cline{3-8}
 & & & 5 & 5 & & & \\
 & & - & 5 & 4 & & & \\ \cline{4-8}
 & & & & 1 & 8 & & \\
 & & & - & 1 & 8 & & \\ \cline{5-8}
 & & & & & 0 & 5 & \\
 & & & & - & & 0 & \\ \cline{6-8}
 & & & & & & 5 & 5 \\
 & & & & & - & 5 & 4 \\ \cline{7-8}
 & & & & & & & 1
\end{array}$$

$$\frac{1\,855\,855}{9} = \frac{206\,206 \times 9}{9} + \frac{1}{9}$$
$$= 206\,206\frac{1}{9}$$

The block of digits 206 repeats because the long division algorithm leads us to perform the same division over and over again. In the algorithm shown above, we see that there are two groups of 9 in 18, leaving a remainder of 0. When we bring down the 5, we see that there are zero groups of 9 in 5. When we bring down the next 5, we see that there are six groups of 9 in 55, leaving a remainder of 1. That is when the process starts over because the next step is to bring down 8, giving us 18, which is what we started with. Since the division repeats, then the digits in the quotient will repeat.

4. Is the number $\frac{15}{11} = 1.36363636...$ rational? Explain.

The number appears to be rational because the decimal expansion has a repeat block of 36. Because every rational number has a block that repeats, the number is rational.

I can rewrite $\frac{15}{11} = 1.\overline{36}$ because the decimal expansion has a repeating block of 36.

5. Is the number $\sqrt{5} = 2.23606787...$ rational? Explain.

The number appears to have a decimal expansion that does not have decimal digits that repeat in a block. For that reason, this is not a rational number.

I don't see any pattern to the decimal expansion.

G8-M7-Lesson 9: Decimal Expansions of Fractions, Part 1

It is better to use a higher power of 10; extra zeros will not change the value of the decimal expansion.

1. Choose a power of 10 to convert this fraction to a decimal: $\frac{3}{11}$.
   Choices will vary.

   The work shown below uses the factor 10^6. Students should choose a factor of at least 10^4 in order to get an approximate decimal expansion and notice that the decimal expansion repeats.

   a. Determine the decimal expansion of $\frac{3}{11}$. Verify you are correct using a calculator.

$$\frac{3}{11} = \frac{3 \times 10^6}{11} \times \frac{1}{10^6}$$
$$= \frac{3\,000\,000}{11} \times \frac{1}{10^6}$$

$$3\,000\,000 = 272\,727 \times 11 + 3$$

I can use division with remainder to find that $3{,}000{,}000 = 272{,}727 \times 11 + 3$.

$$\frac{3}{11} = \frac{272\,727 \times 11 + 3}{11} \times \frac{1}{10^6}$$
$$= \left(\frac{272\,727 \times 11}{11} + \frac{3}{11}\right) \times \frac{1}{10^6}$$
$$= \left(272\,727 + \frac{3}{11}\right) \times \frac{1}{10^6}$$
$$= 272\,727 \times \frac{1}{10^6} + \left(\frac{3}{11} \times \frac{1}{10^6}\right)$$
$$= \frac{272\,727}{10^6} + \left(\frac{3}{11} \times \frac{1}{10^6}\right)$$
$$= 0.272\,727 + \left(\frac{3}{11} \times \frac{1}{10^6}\right)$$

The decimal expansion of $\frac{3}{11}$ is approximately 0.272727.

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2. Ted wrote the decimal expansion of $\frac{5}{7}$ as 0.724285, but when he checked it on a calculator, it was 0.714285. Identify the error from his work below and explain what he did wrong.

$$\frac{5}{7} = \frac{5 \times 10^6}{7} \times \frac{1}{10^6}$$
$$= \frac{5\,000\,000}{7} \times \frac{1}{10^6}$$
$$5\,000\,000 = 724\,285 \times 7 + 5$$

$$\frac{5}{7} = \frac{724\,285 \times 7 + 5}{7} \times \frac{1}{10^6}$$
$$= \left(\frac{724\,285 \times 7}{7} + \frac{5}{7}\right) \times \frac{1}{10^6}$$
$$= \left(724\,285 + \frac{5}{7}\right) \times \frac{1}{10^6}$$
$$= 724\,285 \times \frac{1}{10^6} + \left(\frac{5}{7} \times \frac{1}{10^6}\right)$$
$$= \frac{724\,285}{10^6} + \left(\frac{5}{7} \times \frac{1}{10^6}\right)$$
$$= 0.724285 + \left(\frac{5}{7} \times \frac{1}{10^6}\right)$$

Ted did the division with remainder incorrectly. He wrote that $5,000,000 = 724,285 \times 7 + 5$ when actually $5,000,000 = 714,285 \times 7 + 5$. This error led to his decimal expansion being incorrect.

This is such a small value that it won't affect the estimate. In fact, $\left(\frac{4}{7} \times \frac{1}{10^6}\right) < 0.000001$.

3. Given that $\frac{4}{7} = 0.571428 + \left(\frac{4}{7} \times \frac{1}{10^6}\right)$, explain why 0.571248 is a good estimate of $\frac{4}{7}$.

When you consider the value of $\left(\frac{4}{7} \times \frac{1}{10^6}\right)$, then it is clear that 0.571428 is a good estimate of $\frac{4}{7}$. We know that $\frac{4}{7} < 1$. By the basic inequality property, we also know that $\frac{4}{7} \times \frac{1}{10^6} < 1 \times \frac{1}{10^6}$, which means that $\frac{4}{7} \times \frac{1}{10^6} < 0.000001$. That is such a small value that it will not affect the estimate of $\frac{4}{7}$ in any real way.

G8-M7-Lesson 10: Converting Repeating Decimals to Fractions

1. Let $x = 0.\overline{912}$. Explain why multiplying both sides of this equation by 10^3 will help us determine the fractional representation of x.

When we multiply both sides of the equation by 10^3, on the right side we will have $912.912912\ldots$. This is helpful because we will be able to subtract the repeating decimal from both sides by subtracting x.

I want to multiply x by 10^n, where n is the number of digits in the repeating block, so that the remaining decimal digits are those that repeat.

a. After multiplying both sides of the equation by 10^3, rewrite the resulting equation by making a substitution that will help determine the fractional value of x. Explain how you were able to make the substitution.

$$x = 0.\overline{912}$$
$$(10^3)x = (10^3)(0.\overline{912})$$
$$1000x = 912.\overline{912}$$
$$1000x = 912 + 0.912\,912\ldots$$
$$1000x = 912 + x$$

When I write $912.\overline{912}$ as a sum, I see that $0.\overline{912} = x$, and I can use substitution.

Since we let $x = 0.\overline{912}$, we can substitute the repeating decimal $0.912912\ldots$ with x.

b. Solve the equation to determine the value of x.

$$1000x - x = 912 + x - x$$
$$999x = 912$$
$$\frac{999x}{999} = \frac{912}{999}$$
$$x = \frac{912}{999}$$

I can use properties of equality to solve for x.

c. Is your answer reasonable? Check your answer using a calculator.

Yes, my answer is reasonable and correct. It is reasonable because the denominator cannot be expressed as a product of 2's and 5's; therefore, I know that the fraction must represent an infinite decimal. Also, the number 0.912 is closer to 1 than 0.5, and the fraction is also closer to 1 than $\frac{1}{2}$. It is correct because the division of $\frac{912}{999}$ using the calculator is $0.912912912\ldots$.

2. Find the fraction equal to $6.7\overline{56}$. Check your answer using a calculator.

I need to let y equal the repeating block of my decimal. I will repeat the process I learned in class with both x and y.

Let $x = 6.7\overline{56}$.

$$x = 6.7\overline{56}$$
$$10x = (10)6.7\overline{56}$$
$$10x = 67.\overline{56}$$

I am going to multiply by 10 so that the only remaining decimal digits are those that repeat.

Let $y = 0.\overline{56}$.

$$y = 0.\overline{56}$$
$$100y = 100(0.\overline{56})$$
$$100y = 56.\overline{56}$$
$$100y = 56 + y$$
$$100y - y = 56 + y - y$$
$$99y = 56$$
$$\frac{99y}{99} = \frac{56}{99}$$
$$y = \frac{56}{99}$$

$$10x = 67.\overline{56}$$
$$10x = 67 + y$$
$$10x = 67 + \frac{56}{99}$$
$$10x = \frac{67 \times 99}{99} + \frac{56}{99}$$
$$10x = \frac{67 \times 99 + 56}{99}$$
$$10x = \frac{6689}{99}$$
$$\left(\frac{1}{10}\right)10x = \left(\frac{1}{10}\right)\frac{6689}{99}$$
$$x = \frac{6689}{990}$$

$$6.7\overline{56} = \frac{6689}{990}$$

G8-M7-Lesson 11: The Decimal Expansion of Some Irrational Numbers

1. Use the method of rational approximation to determine the decimal expansion of $\sqrt{42}$. Determine which interval of hundredths it would lie in.

   The number $\sqrt{42}$ is between 6 and 7 but closer to 6.

   Looking at the interval of tenths, beginning with 6.4 to 6.5, the number $\sqrt{42}$ lies between 6.4 and 6.5 because $6.4^2 = 40.96$ and $6.5^2 = 42.25$ but is closer to 6.5.

   In the interval of hundredths, the number $\sqrt{42}$ lies between 6.48 and 6.49 because $6.48^2 = 41.9904$ and $6.49^2 = 42.1201$.

   Since 42 is closer to 6.48^2 than 6.49^2, the number $\sqrt{42}$ is approximately 6.48.

   In class, I learned rational approximation by examining the number at increasingly smaller powers such as tenths, hundredths, and so on.

   I know $\sqrt{42}$ is between 6 and 7 because $6^2 < \left(\sqrt{42}\right)^2 < 7^2$. Since $\sqrt{42}$ is almost halfway between $\sqrt{36}$ and $\sqrt{49}$, I will start looking at 6.4.

2. Determine the three-decimal digit approximation of the number $\sqrt{83}$.

   The number $\sqrt{83}$ is between 9 and 10 but closer to 9.

   Looking at the interval of tenths, beginning with 9.1 to 9.2, the number $\sqrt{83}$ lies between 9.1 and 9.2 because $9.1^2 = 82.81$, and $9.2^2 = 84.64$ and is closer to 9.1.

   In the interval of hundredths, the number $\sqrt{83}$ lies between 9.11 and 9.12 because $9.11^2 = 82.9921$, and $9.12^2 = 83.1744$ and is closer to 9.11.

   So far my decimal approximation is $\sqrt{83} \approx 9.11$. But I need to continue this process for one more decimal place.

   In the interval of thousandths, the number $\sqrt{83}$ lies between 9.110 and 9.111 because $9.110^2 = 82.9921$, and $9.111^2 = 83.010321$ but is closer to 82.9921.

   Since 83 is closer to 9.110^2 than 9.111^2, the three-decimal digit approximation of the number is approximately 9.110.

3. Is the number $\sqrt{360}$ rational or irrational? Explain your answer.

   The number $\sqrt{360}$ is an irrational number because it has a decimal expansion that is infinite and does not repeat. That is, the number $\sqrt{360}$ cannot be expressed as a rational number; therefore, it is irrational.

I learned that irrational numbers are infinite decimals that do not have repeating blocks of digits.

4. Is the number $0.94949494...$ rational or irrational? Explain your answer.

   The number $0.94949494...$ can be expressed as the fraction $\frac{94}{99}$; therefore, it is a rational number. Not only is the number $\frac{94}{99}$ a quotient of integers, but its decimal expansion is infinite with a repeating block of digits.

I learned how to convert repeating decimals to fractions in the previous lesson. Repeating blocks of digits means this is a rational number.

5. Challenge: Determine the two-decimal digit approximation of the number $\sqrt[3]{25}$.

   The number $\sqrt[3]{25}$ is between integers 2 and 3 because $2^3 < \left(\sqrt[3]{25}\right)^3 < 3^3$, or $8 < 25 < 27$.

   Since $\sqrt[3]{25}$ is closer to 3, I will start checking the tenths intervals between 2.9 and 3. $\sqrt[3]{25}$ is between 2.9 and 3 since $2.9^3 = 24.389$ and $3^3 = 27$.

   Checking the hundredths interval, $\sqrt[3]{25}$ is between 2.92 and 2.93 since $2.92^3 = 24.897088$ and $2.93^3 = 25.153757$.

   Since 25 is closer to 2.92^3 than 2.93^3, the two-decimal digit approximation is 2.92.

When I use rational approximation, I need to cube the values instead of squaring them since I am trying to approximate the cube root of 25.

G8-M7-Lesson 12: Decimal Expansions of Fractions, Part 2

1. Use rational approximation to determine the decimal expansion of $\frac{14}{9}$.

$$\frac{14}{9} = \frac{9}{9} + \frac{5}{9}$$
$$= 1 + \frac{5}{9}$$

I know that $\frac{14}{9}$ lies between integers 1 and 2.

The ones digit is 1.

In the interval of tenths, we are looking for integers m *and* $m+1$ *so that*

$$\frac{m}{10} < \frac{5}{9} < \frac{m+1}{10},$$

which is the same as

$$m < \frac{50}{9} < m+1.$$

Since I am looking for the tenths digit, I will use the denominator of 10 with my consecutive numerators of m and $m+1$.

Multiply every term by 10.

$$\frac{50}{9} = \frac{45}{9} + \frac{5}{9}$$
$$= 5 + \frac{5}{9}$$

The tenths digit is 5. *The difference between* $\frac{5}{9}$ *and* $\frac{5}{10}$ *is*

$$\frac{5}{9} - \frac{5}{10} = \frac{5}{90}.$$

In the interval of hundredths, we are looking for integers m *and* $m+1$ *so that*

$$\frac{m}{100} < \frac{5}{90} < \frac{m+1}{100},$$

which is the same as

$$m < \frac{500}{90} < m+1.$$

I already did this when I was finding the tenths digit.

However, we already know that $\frac{500}{90} = \frac{50}{9} = 5 + \frac{5}{9}$; *therefore, the hundredths digit is* 5. *Because we keep getting* $\frac{5}{9}$, *we can assume the digit of* 5 *will continue to repeat. Therefore, the decimal expansion of* $\frac{14}{9}$ *is* $1.555\ldots$.

2. Use rational approximation to determine the decimal expansion of $\frac{83}{37}$ to at least 3 decimal digits.

This problem is asking for the decimal expansion of $\frac{9}{37}$.

$$\frac{83}{37}=\frac{74}{37}+\frac{9}{37}$$
$$=2+\frac{9}{37}$$

I will use the same process as Problem 1 to find the tenths, hundredths, and thousandths digits.

The ones digit is 2.

In the interval of tenths, we are looking for integers m *and* $m+1$ *so that*

$$\frac{m}{10}<\frac{9}{37}<\frac{m+1}{10},$$

which is the same as

$$m<\frac{90}{37}<m+1.$$

$$\frac{90}{37}=\frac{74}{37}+\frac{16}{37}=2+\frac{16}{37}$$

So far, I have 2.2 as my decimal expansion.

The tenths digit is 2.

The difference between $\frac{9}{37}$ *and* $\frac{2}{10}$ *is*

$$\frac{9}{37}-\frac{2}{10}=\frac{16}{370}.$$

I need to find the difference between $\frac{9}{37}$ and 0.2.

In the interval of hundredths, we are looking for integers m *and* $m+1$ *so that*

$$\frac{m}{100}<\frac{16}{370}<\frac{m+1}{100},$$

which is the same as

$$m<\frac{1600}{370}<m+1.$$

$$\frac{1600}{370}=\frac{160}{37}=\frac{148}{37}+\frac{12}{37}=4+\frac{12}{37}$$

So far, I have 2.24 as my decimal expansion.

The hundredths digit is 4.

The difference between $\frac{9}{37}$ *and* $\left(\frac{2}{10}+\frac{4}{100}\right)$ *is*

$$\frac{9}{37}-\left(\frac{2}{10}+\frac{4}{100}\right)=\frac{9}{37}-\frac{24}{100}=\frac{12}{3700}.$$

I need to find the difference between $\frac{9}{37}$ and the tenths and hundredths digits and use the result to find the thousandths digit.

In the interval of thousandths, we are looking for integers m and $m+1$ so that

$$\frac{m}{1000}<\frac{12}{3700}<\frac{m+1}{1000},$$

which is the same as

$$m<\frac{12000}{3700}<m+1.$$

$$\frac{12000}{3700}=\frac{120}{37}=\frac{111}{37}+\frac{9}{37}=3+\frac{9}{37}$$

The fraction $\frac{9}{37}$ was where I started. The decimals will repeat from this point on.

The thousandths digit is 3.

We see again the fraction $\frac{9}{37}$, so we can expect the decimal digits to repeat at this point. Therefore, the decimal approximation of $\frac{83}{37}$ is 2.243243243....

3. Use rational approximation to determine which number is larger, $\sqrt{12}$ or $\frac{11}{3}$.

The number $\sqrt{12}$ is between 3 and 4. In the sequence of tenths, $\sqrt{12}$ is between 3.4 and 3.5 because $3.4^2<\left(\sqrt{12}\right)^2<3.5^2$. In the sequence of hundredths, $\sqrt{12}$ is between 3.46 and 3.47 because $3.46^2<\left(\sqrt{12}\right)^2<3.47^2$. In the sequence of thousandths, $\sqrt{12}$ is between 3.464 and 3.465 because $3.464^2<\left(\sqrt{12}\right)^2<3.465^2$. The decimal expansion of $\sqrt{12}$ is approximately 3.464....

$$\frac{11}{3}=\frac{9}{3}+\frac{2}{3}=3+\frac{2}{3}$$

I remember this type of problem from the last lesson.

In the interval of tenths, we are looking for the integers m and $m+1$ so that

$$\frac{m}{10}<\frac{2}{3}<\frac{m+1}{10},$$

which is the same as

$$m<\frac{20}{3}<m+1.$$

$$\frac{20}{3}=\frac{18}{3}+\frac{2}{3}=6+\frac{2}{3}$$

The tenths digit is 6. Since the fraction $\frac{2}{3}$ has reappeared, we can assume that the next digit is also 6, and the work will continue to repeat. Therefore, the decimal expansion of $\frac{11}{3}$ is 3.666..., and $\frac{11}{3}>\sqrt{12}$.

G8-M7-Lesson 13: Comparing Irrational Numbers

1. Which number is smaller, $\sqrt[3]{125}$ or $\sqrt{30}$? Explain your answer.

I can use what I know about perfect squares to approximate $\sqrt{30}$.

$$\sqrt[3]{125} = \sqrt[3]{5^3} = 5$$

The number $\sqrt{30}$ is between 5 and 6 but definitely more than 5. Therefore, $\sqrt[3]{125} < \sqrt{30}$, and $\sqrt[3]{125}$ is smaller.

2. Which number is smaller, $\sqrt{64}$ or $\sqrt[3]{512}$? Explain your answer.

$$\sqrt{64} = \sqrt{8^2} = 8$$
$$\sqrt[3]{512} = \sqrt[3]{8^3} = 8$$

The numbers $\sqrt{64}$ and $\sqrt[3]{512}$ are equal because both are equal to 8.

3. Which number is larger, $\sqrt{68}$ or $\frac{829}{99}$? Explain your answer.

I can use the method of rational approximation or long division to find the decimal expansion of $\frac{829}{99}$.

The number $\frac{829}{99}$ is equal to $8.\overline{37}$.

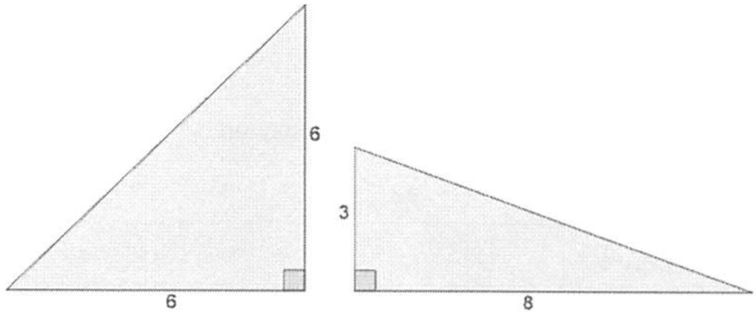
The number $\sqrt{68}$ is between 8 and 9 because $8^2 < \left(\sqrt{68}\right)^2 < 9^2$. The number $\sqrt{68}$ is between 8.2 and 8.3 because $8.2^2 < \left(\sqrt{68}\right)^2 < 8.3^2$. The approximate decimal value of $\sqrt{68}$ is 8.2…. Since $8.2 < 8.\overline{37}$, then $\sqrt{68} < \frac{829}{99}$. Thus, the fraction $\frac{829}{99}$ is larger.

I only need to determine the decimal approximation for the tenths digit of $\sqrt{68}$ to determine which number is larger.

4. Which number is larger, $\frac{11}{15}$ or $0.\overline{732}$? Explain your answer.

The number $\frac{11}{15}$ is equal to $0.7\overline{3}$. Since $0.7\overline{3} > 0.\overline{732}$…, then $\frac{11}{15} > 0.\overline{732}$.
Thus, the number $\frac{11}{15}$ is larger.

5. Which of the two right triangles shown below, measured in units, has the longer hypotenuse? Approximately how much longer is it?



Let x represent the length of the hypotenuse of the triangle on the left.

$$6^2 + 6^2 = x^2$$
$$36 + 36 = x^2$$
$$72 = x^2$$
$$\sqrt{72} = \sqrt{x^2}$$
$$\sqrt{72} = x$$

I need to use two different variables since I don't know the value of either or if they are the same number.

The number $\sqrt{72}$ is between 8 and 9 because $8^2 < \left(\sqrt{72}\right)^2 < 9^2$. The number $\sqrt{72}$ is between 8.4 and 8.5 because $8.4^2 < \left(\sqrt{72}\right)^2 < 8.5^2$. The number $\sqrt{72}$ is between 8.48 and 8.49 because $8.48^2 < \left(\sqrt{72}\right)^2 < 8.49^2$. The approximate decimal value of $\sqrt{72}$ is 8.48....

Let y represent the length of the hypotenuse of the triangle on the right.

$$3^2 + 8^2 = y^2$$
$$9 + 64 = y^2$$
$$73 = y^2$$
$$\sqrt{73} = \sqrt{y^2}$$
$$\sqrt{73} = y$$

I know that $\sqrt{72} < \sqrt{73}$, but the question is asking approximately how much longer is $\sqrt{73}$ than $\sqrt{72}$, so I need to determine the decimal approximation for y.

The number $\sqrt{73}$ is between 8 and 9 because $8^2 < \left(\sqrt{73}\right)^2 < 9^2$. The number $\sqrt{73}$ is between 8.5 and 8.6 because $8.5^2 < \left(\sqrt{73}\right)^2 < 8.6^2$. The number $\sqrt{73}$ is between 8.54 and 8.55 because $8.54^2 < \left(\sqrt{73}\right)^2 < 8.55^2$. The approximate decimal value of $\sqrt{73}$ is 8.54....

The triangle on the right has the longer hypotenuse. It is approximately 0.06 units longer than the hypotenuse of the triangle on the left.

EUREKA MATH

G8-M7-Lesson 14: Decimal Expansion of π

1. Carrie estimated π to be $3.10 < \pi < 3.21$. If she uses this approximation of π to determine the area of a circle with a radius of 7 in., what could the area be?

   The area of the circle with radius 7 in. *will be between* 151.9 in^2 *and* 157.29 in^2.

   I need to substitute the values for π and the radius into the formula for the area of a circle, namely $A = \pi r^2$.

2. Winston estimated the circumference of a circle with a radius of 8.3 in. to be 52.29 in. What approximate value of π did he use? Is it an acceptable approximation of π? Explain.

   $$C = 2\pi r$$
   $$52.29 = 2\pi(8.3)$$
   $$52.29 = 16.6\pi$$
   $$\frac{52.29}{16.6} = \pi$$
   $$3.15 = \pi$$

   I need to solve for π using properties of equality.

   Winston used 3.15 *to approximate* π. *This is an acceptable approximation for* π *because it is in the interval that we approximated* π *to be in the lesson,* $3.10 < \pi < 3.21$.

3. Estimate the value of the irrational number $(2.3856\ldots)^2$.

   I need to add 0.0001 to 2.3856 and then square it.

   $$2.3856^2 < (2.3856\ldots)^2 < 2.3857^2$$
   $$5.691\,087\,36 < (2.3856\ldots)^2 < 5.691\,564\,49$$

   $(2.3856\ldots)^2 = 5.691$ *is correct up to three decimal digits.*

4. Estimate the value of the irrational number $(0.956321\ldots)^2$.

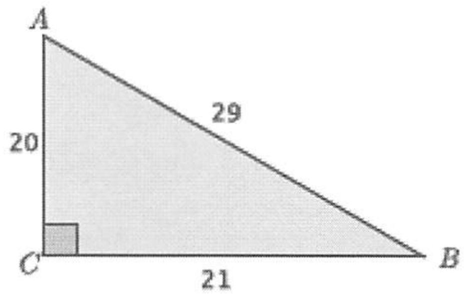
   $$0.956\,321^2 < (0.956\,321\ldots)^2 < 0.956\,322^2$$
   $$0.914\,549\,855 < (0.956\,321\ldots)^2 < 0.914\,551\,767\,7$$

   $(0.956321\ldots)^2 = 0.9145$ *is correct up to four decimal digits.*

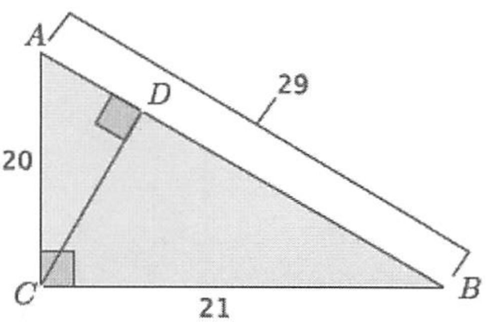
   I need to count how many decimal places the values are exactly the same.

G8-M7-Lesson 15: Pythagorean Theorem, Revisited

1. For the right triangle shown below, identify and use similar triangles to illustrate the Pythagorean theorem.

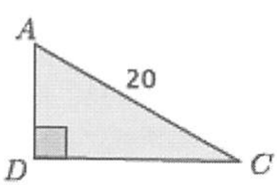


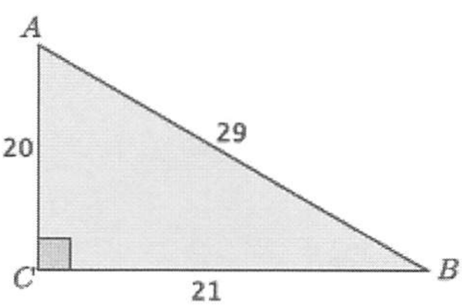
First, draw a segment that is perpendicular to side $\overline{AB}$ that goes through point C. The point of intersection of that segment and side $\overline{AB}$ will be marked as point D.

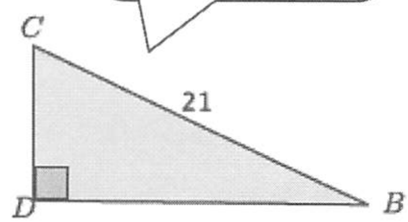


Then, I have three similar triangles, $\triangle ABC$, $\triangle CBD$, and $\triangle ACD$, as shown below.

Using a sequence of transformations, I can see each right triangle in the same orientation.







$\triangle ABC$ and $\triangle CBD$ are similar because each one has a right angle, and they both share $\angle B$. By AA criterion, $\triangle ABC \sim \triangle CBD$.

$\triangle ABC$ and $\triangle ACD$ are similar because each one has a right angle, and they both share $\angle A$. By AA criterion, $\triangle ABC \sim \triangle ACD$.

By the transitive property, we also know that $\triangle ACD \sim \triangle CBD$.

Since the triangles are similar, they have corresponding sides that are equal in ratio.

For $\triangle ABC$ *and* $\triangle CBD$,

$$\frac{21}{29} = \frac{|BD|}{21},$$

which is the same as $21^2 = 29(|BD|)$.

For $\triangle ABC$ *and* $\triangle ACD$,

$$\frac{20}{29} = \frac{|AD|}{20},$$

which is the same as $20^2 = 29(|AD|)$.

Because my right triangles are in the same orientation, I can make the following proportion using $\triangle ABC$ and $\triangle CBD$:
$\frac{\text{base}}{\text{hypotenuse}} = \frac{\text{base}}{\text{hypotenuse}}$.
I can solve any proportion by using cross multiplication.

Adding these two equations together, I get

$$21^2 + 20^2 = 29(|BD|) + 29(|AD|).$$

By the distributive property,

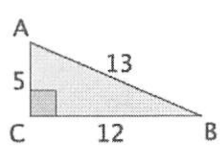
$$21^2 + 20^2 = 29(|BD| + |AD|);$$

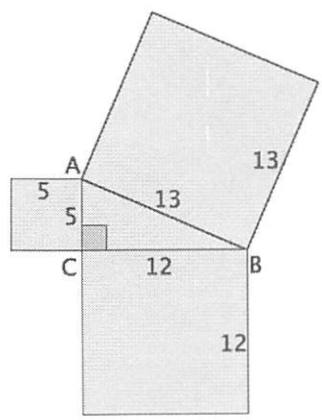
however, $|BD| + |AD| = |AB| = 29$. *Therefore,*

$$20^2 + 21^2 = 29(29)$$
$$20^2 + 21^2 = 29^2.$$

I found this by looking at my original triangle and the triangle where the altitude was drawn.

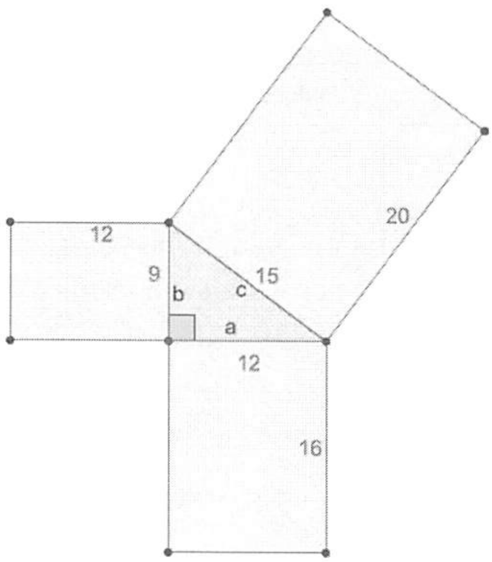
2. For the right triangle shown below, identify and use squares formed by the sides of the triangle to illustrate the Pythagorean theorem.





The sum of the areas of the smallest squares is $5^2 + 12^2$*, which is* 169. *The area of the largest square is* 13^2*, which is* 169. *The sum of the areas of the squares of the legs is equal to the area of the square of the hypotenuse; therefore, for legs* a *and* b*, and hypotenuse* c*, we see that* $a^2 + b^2 = c^2$.

3. Can any similar figures be drawn off the sides of the right triangle to prove the Pythagorean theorem? Use computations to show that the sum of the areas of the figures off of the sides a and b equals the area of the figure off of side c.



The rectangles are similar because their corresponding side lengths are equal in scale factor.

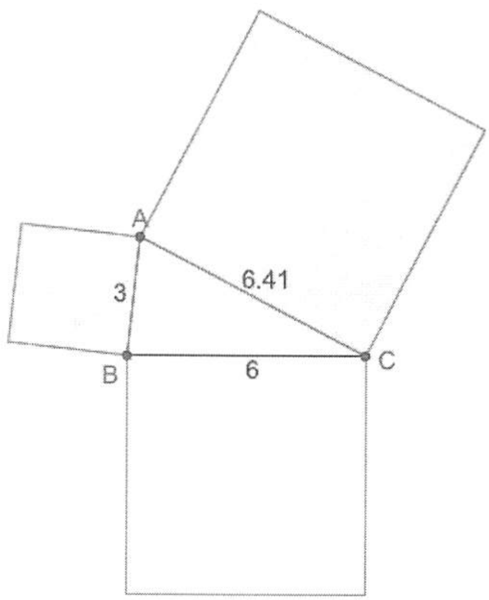
$$\frac{9}{12}=\frac{12}{16}=\frac{15}{20}=0.75$$

The area of the smaller rectangles are 108 square units and 192 square units, and the area of the larger rectangle is 300 square units. The sum of the smaller areas is equal to the larger area.

$$108+192=300$$
$$300=300$$

Therefore, the sum of the areas of the smaller similar rectangles does equal the area of the larger similar rectangle proving the Pythagorean theorem with similar figures.

4. The following image for the Pythagorean theorem contains an error. Explain what is wrong.



Based on the proof shown in class, we would expect the sum of the two smaller areas to be equal to the larger area. The smaller areas are 9 and 36, while the larger area is 41.0881. That is, $9+36$ should equal 41.0881. However, $9+36=45$.

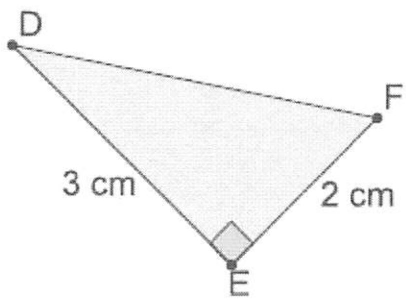
We know that the Pythagorean theorem only works for right triangles. Considering the converse of the Pythagorean theorem, when we use the given side lengths, we do not get a true statement.

$$3^2+6^2=6.41^2$$
$$9+36=41.0881$$
$$45\neq 41.0881$$

Therefore, the original triangle is not a right triangle, so it makes sense that the areas of the squares were not adding up like we expected.

G8-M7-Lesson 16: Converse of the Pythagorean Theorem

1. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.



Let c cm represent the hypotenuse of the triangle.

$$3^2 + 2^2 = c^2$$
$$9 + 4 = c^2$$
$$13 = c^2$$
$$\sqrt{13} = \sqrt{c^2}$$
$$3.6 \approx c$$

To estimate, I need the two perfect squares that surround 13, which are 9 and 16. 13 is slightly closer to 16 than 9, so $\sqrt{13}$ is closer to $\sqrt{16} = 4$ than $\sqrt{9} = 3$.

The hypotenuse of the triangle is exactly $\sqrt{13}$ cm and approximately 3.6 cm.

2. Is the triangle with leg lengths of $\sqrt{5}$ cm and 7 cm and hypotenuse of length $\sqrt{54}$ cm a right triangle? Show your work, and answer in a complete sentence.

$$\left(\sqrt{5}\right)^2 + 7^2 = \left(\sqrt{54}\right)^2$$
$$5 + 49 = 54$$
$$54 = 54$$

To simplify a square root that is squared, I need to remember the following:

$$\left(\sqrt{5}\right)^2 = \sqrt{5} \cdot \sqrt{5}$$
$$\left(\sqrt{5}\right)^2 = \sqrt{25}$$
$$\left(\sqrt{5}\right)^2 = 5$$

Yes, the triangle with leg lengths of $\sqrt{5}$ cm and 7 cm and hypotenuse of length $\sqrt{54}$ cm is a right triangle because the lengths satisfy the Pythagorean theorem.

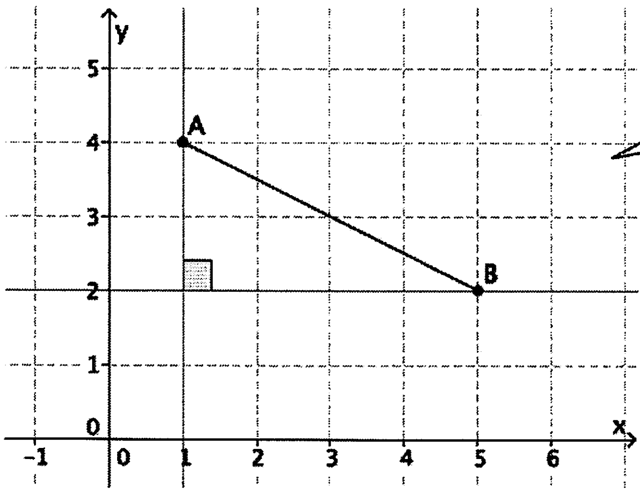
3. Is the triangle with leg lengths of $\sqrt{8}$ cm and 10 cm and hypotenuse of length $\sqrt{164}$ cm a right triangle? Show your work, and answer in a complete sentence.

$$\left(\sqrt{8}\right)^2 + 10^2 = \left(\sqrt{164}\right)^2$$
$$8 + 100 = 164$$
$$108 \neq 164$$

No, the triangle with leg lengths of $\sqrt{8}$ cm and 10 cm and hypotenuse of length $\sqrt{164}$ cm is not a right triangle because the lengths do not satisfy the Pythagorean theorem.

G8-M7-Lesson 17: Distance on the Coordinate Plane

1. Determine the distance between points A and B on the coordinate plane. Round your answer to the tenths place.



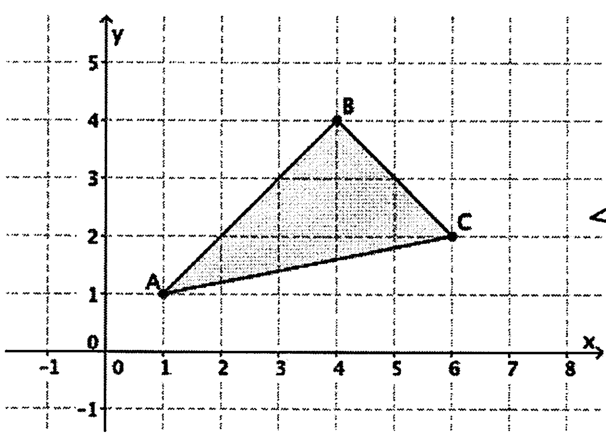
To determine the distance between the points, I am going to use the grid lines from the coordinate plane to create a right triangle.

Let c represent $|AB|$.

$$2^2 + 4^2 = c^2$$
$$4 + 16 = c^2$$
$$20 = c^2$$
$$\sqrt{20} = c$$
$$4.5 \approx c$$

The distance between points A and B is about 4.5 ***units.***

2. Is the triangle formed by points A, B, and C a right triangle?



None of these sides are horizontal or vertical, so I cannot simply count to find their lengths.

I need to create right triangles to find the lengths of the sides and then use those lengths in the Pythagorean theorem.

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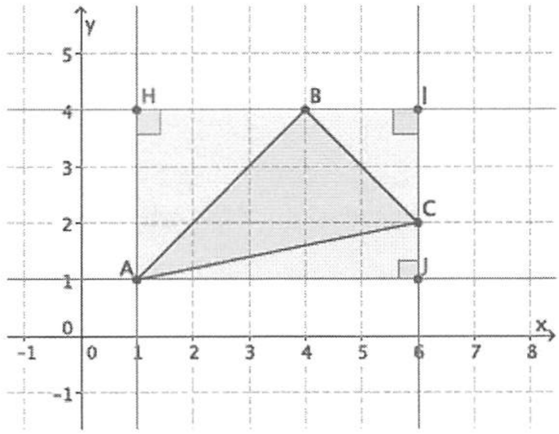


Let c represent $|AB|$.

$$\begin{aligned} 3^2 + 3^2 &= c^2 \\ 9 + 9 &= c^2 \\ 18 &= c^2 \\ \sqrt{18} &= \sqrt{c^2} \\ \sqrt{18} &= c \end{aligned}$$

Let c represent $|BC|$.

$$\begin{aligned} 2^2 + 2^2 &= c^2 \\ 4 + 4 &= c^2 \\ 8 &= c^2 \\ \sqrt{8} &= \sqrt{c^2} \\ \sqrt{8} &= c \end{aligned}$$

Let c represent $|AC|$.

$$\begin{aligned} 5^2 + 1^2 &= c^2 \\ 25 + 1 &= c^2 \\ 26 &= c^2 \\ \sqrt{26} &= \sqrt{c^2} \\ \sqrt{26} &= c \end{aligned}$$

$$\begin{aligned} \left(\sqrt{18}\right)^2 + \left(\sqrt{8}\right)^2 &= \left(\sqrt{26}\right)^2 \\ 18 + 8 &= 26 \\ 26 &= 26 \end{aligned}$$

The hypotenuse is side $\overline{AC}$ since $\sqrt{26}$ is the longest side. I substituted into the Pythagorean theorem and got a true statement, meaning I have a right triangle.

Yes, the points do form a right triangle.

G8-M7-Lesson 18: Applications of the Pythagorean Theorem

1. A 55 in. TV is advertised on sale at a local store. What are the length and width of the television?

   Let x be the factor applied to the ratio $4:3$.

$$\begin{aligned}(4x)^2 + (3x)^2 &= 55^2\\ 16x^2 + 9x^2 &= 3025\\ 25x^2 &= 3025\\ \frac{25}{25}x^2 &= \frac{3025}{25}\\ x^2 &= 121\\ \sqrt{x^2} &= \sqrt{121}\\ x &= 11\end{aligned}$$

I remember from the notes that the dimensions of a television are in the ratio $4:3$ and that the size of the television is actually the length of the diagonal. Therefore, $4x$ and $3x$ are the legs of the right triangle while 55 is the hypotenuse.

Since $x = 11$, $3x = 33$ and $4x = 44$. Therefore, the dimensions of the TV are 44 in. by 33 in.

2. The soccer team was instructed to run the perimeter of their soccer field, which has dimensions of 115 yards by 74 yards. One player decided to run the length and width and then finished by running diagonally. To the nearest tenth of a yard, how many fewer yards of running did this player complete than the rest of the team?

$$\begin{aligned}P &= 2l + 2w\\ P &= 2(115) + 2(74)\\ P &= 230 + 148\\ P &= 378\end{aligned}$$

The team ran 378 yards.

Let a yards represent the length of the field, b yards represent the width of the field, and c yards represent the diagonal length of the field.

$$\begin{aligned}a^2 + b^2 &= c^2\\ 115^2 + 74^2 &= c^2\\ 13225 + 5476 &= c^2\\ 18701 &= c^2\\ \sqrt{18701} &= \sqrt{c^2}\\ 136.8 &\approx c\end{aligned}$$

I need to find the diagonal length of the field and then use it to find the total distance the single player ran. The number $\sqrt{18{,}701}$ is between 136 and 137. In the sequence of tenths, the number is between 136.7 and 136.8 because $136.7^2 < \left(\sqrt{18{,}701}\right)^2 < 136.8^2$. Since 18,701 is closer to 136.8^2 than 136.7^2, the approximate length of the hypotenuse is 136.8 yards.

$$115 + 74 + 136.8 = 325.8$$

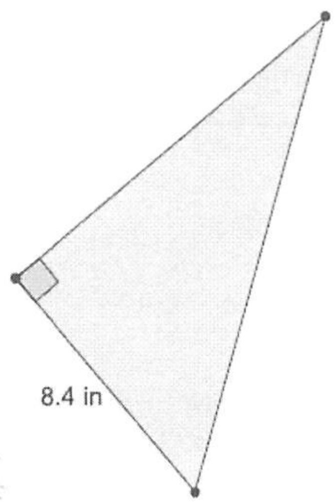
The player ran approximately 325.8 *yards.*

$$378 - 325.8 = 52.2$$

The player ran approximately 52.2 *fewer yards than the rest of the team.*

3. The area of the right triangle shown below is 51.24 in^2.

   a. What is the height of the triangle?



Let h in. *represent the height of the triangle.*

$$A = \frac{1}{2}bh$$
$$51.24 = \frac{1}{2}(8.4)h$$
$$51.24 = 4.2h$$
$$\frac{51.24}{4.2} = \frac{4.2}{4.2}h$$
$$12.2 = h$$

The height of the triangle is 12.2 in.

   b. What is the perimeter of the right triangle? Round your answer to the tenths place.

Let c in. *represent the length of the hypotenuse.*

$$8.4^2 + 12.2^2 = c^2$$
$$70.56 + 148.84 = c^2$$
$$219.4 = c^2$$
$$\sqrt{219.4} = \sqrt{c^2}$$
$$14.8 \approx c$$

In order to find the perimeter, I first need all three sides. To find the hypotenuse, I can use the Pythagorean theorem.

The number $\sqrt{219.4}$ *is between* 14 *and* 15. *In the sequence of tenths, the number is between* 14.8 *and* 14.9 *because* $14.8^2 < \left(\sqrt{219.4}\right)^2 < 14.9^2$. *Since* 219.4 *is closer to* 14.8^2 *than* 14.9^2, *the approximate length of the hypotenuse is* 14.8 in.

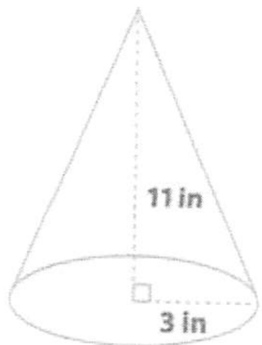
$$8.4 + 12.2 + 14.8 = 35.4$$

The perimeter of the triangle is approximately 35.4 in.

G8-M7-Lesson 19: Cones and Spheres

1. What is the lateral length of the cone shown below? Give an approximate answer rounded to the tenths place.

   Let c in. be the lateral length in inches.

   $$11^2 + 3^2 = c^2$$
   $$121 + 9 = c^2$$
   $$130 = c^2$$
   $$\sqrt{130} = \sqrt{c^2}$$
   $$\sqrt{130} = c$$

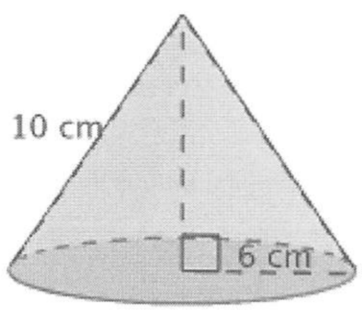
   I know the slanted part of the cone is called the lateral length. I can use the Pythagorean theorem to find its length.

   The number $\sqrt{130}$ is between 11 and 12. In the sequence of tenths, it is between 11.4 and 11.5. Since 130 is closer to 11.4^2 than 11.5^2, the approximate value of the number is 11.4.

   The lateral length of the cone is approximately 11.4 in.

2. The cone below has a base radius of 6 cm in length and a lateral length of 10 cm. What is the volume of the cone? Give an exact answer.

   Let h cm represent the height of the cone.

   $$6^2 + h^2 = 10^2$$
   $$36 + h^2 = 100$$
   $$h^2 = 64$$
   $$\sqrt{h^2} = \sqrt{64}$$
   $$h = 8$$

   The height of the cone is 8 cm.

   $$V = \frac{1}{3}\pi(36)(8)$$
   $$= 96\pi$$

   I can use the Pythagorean theorem to find the height of the cone. The volume of the cone is $\frac{1}{3}Bh$. B is the area of the base, which is a circle. I will not estimate π since the problem asked for an exact answer.

   The volume of the cone is 96π cm^3.

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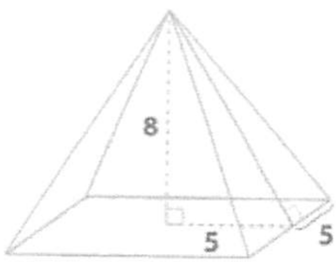
3. Determine the volume and surface area of the pyramid shown below. Give exact answers.



$$V = \frac{1}{3}(100)(8)$$
$$= \frac{800}{3}$$

The volume of a pyramid is the same as the volume of a cone, which is $\frac{1}{3}Bh$. B represents the area of the base.

The volume of the pyramid is $\frac{800}{3}$ *cubic units.*

Let c *units represent the lateral length.*

The base is the shape of a square with side lengths of 10. The four sides (faces) are the shape of triangles with base length of 10.

$$5^2 + 8^2 = c^2$$
$$25 + 64 = c^2$$
$$89 = c^2$$
$$\sqrt{89} = \sqrt{c^2}$$
$$\sqrt{89} = c$$

To find the surface area, I need to sum the areas of all the faces and the base of the pyramid.

The area of one face is $\frac{10\sqrt{89}}{2}$ *square units, which is equal to* $5\sqrt{89}$ *square units. Since there are four faces, the total area is* $4 \times 5\sqrt{89}$ *square units, which is equal to* $20\sqrt{89}$ *square units.*

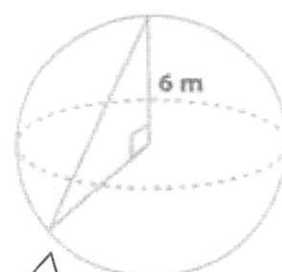
The base area is 100 *square units, and the total area of the faces is* $20\sqrt{89}$ *square units, so the surface area of the pyramid is* $100 + 20\sqrt{89}$ *square units.*

4. What is the length of the chord of the sphere shown below? Give an exact answer using a square root.

Let c m *represent the length of the chord.*

A chord is a segment that connects any two points on a circle or sphere. The chord in the diagram is the hypotenuse of the right triangle.

$$6^2 + 6^2 = c^2$$
$$36 + 36 = c^2$$
$$72 = c^2$$
$$\sqrt{72} = \sqrt{c^2}$$
$$\sqrt{72} = c$$
$$\sqrt{6^2 \times 2} = c$$
$$6\sqrt{2} = c$$

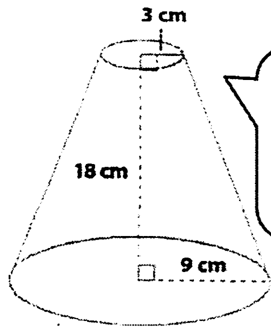


The sides of the triangle are the same measurement since they are both radii of the sphere.

The length of the chord is $6\sqrt{2}$ m.

G8-M7-Lesson 20: Truncated Cones

1. Find the volume of the truncated cone.

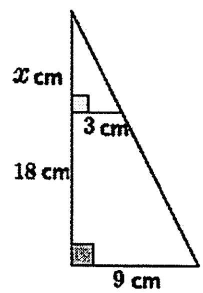


To find the volume of a truncated cone, I must first determine the height of the portion of the cone that has been removed.

a. Write a proportion that will allow you to determine the height of the cone that has been removed. Explain what each part of the proportion represents.

$$\frac{3}{9} = \frac{x}{x+18}$$

I can use ratios that represent the corresponding sides of right triangles. Pictorially it looks like this:



Let x cm represent the height of the small cone. Then x cm + 18 cm is the height of the larger cone. 3 cm is the base radius of the small cone, and 9 cm is the base radius of the large cone.

b. Solve your proportion to determine the height of the cone that has been removed.

$$3(x+18) = 9x$$
$$3x + 54 = 9x$$
$$54 = 6x$$
$$9 = x$$

This means the height of the cone before the smaller portion was removed was 27 cm, since $9 + 18 = 27$.

c. Show a fact about the volume of the truncated cone using an expression. Explain what each part of the expression represents.

$$\frac{1}{3}\pi(9)^2(27) - \frac{1}{3}\pi(3)^2(9)$$

I need to determine the difference between the volume of the large cone and the volume of the small cone.

The expression $\frac{1}{3}\pi(9)^2(27)$ represents the volume of the large cone, and $\frac{1}{3}\pi(3)^2(9)$ is the volume of the small cone. The difference in volumes gives the volume of the truncated cone.

d. Calculate the volume of the truncated cone.

Volume of the small cone:

$$V = \frac{1}{3}\pi(3)^2(9)$$
$$= 27\pi$$

Volume of the large cone:

$$V = \frac{1}{3}\pi(9)^2(27)$$
$$= 729\pi$$

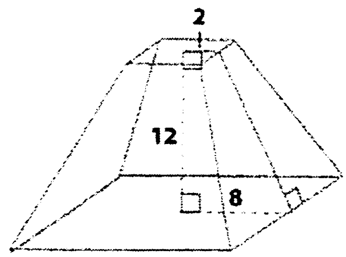
Volume of the truncated cone:

$$729\pi - 27\pi = 702\pi$$

The volume of the truncated cone is $702\pi\ \text{cm}^3$.

2. Find the volume of the truncated pyramid with a square base.

I can use the same process I used to determine the volume of a truncated cone to determine the volume of a truncated pyramid.



Let x units represent the height of the small pyramid.

$$\frac{2}{8} = \frac{x}{x+12}$$
$$2(x+12) = 8x$$
$$2x + 24 = 8x$$
$$24 = 6x$$
$$4 = x$$

I learned how to determine the volumes of pyramids in the last lesson.

Volume of the small pyramid:

$$V = \frac{1}{3}(16)(4)$$
$$= \frac{64}{3}$$

Volume of the large pyramid:

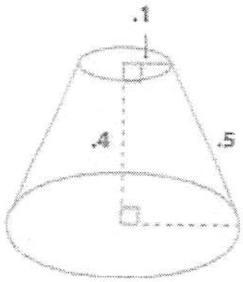
$$V = \frac{1}{3}(256)(16)$$
$$= \frac{4096}{3}$$

Volume of the truncated pyramid:

$$\frac{4096}{3} - \frac{64}{3} = \frac{4032}{3}$$

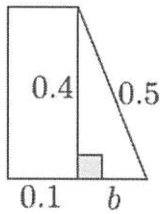
The volume of the truncated pyramid is $\frac{4032}{3}$ *cubic units.*

3. Challenge: Find the volume of the truncated cone.



Since the height of the truncated cone is 0.4 units, we can drop a perpendicular line from the top of the cone to the bottom of the cone so that we have a right triangle with a leg length of 0.4 units and a hypotenuse of 0.5 units. By the Pythagorean theorem, if b units represents the length of the leg of the right triangle, then

I need to determine the radius of the larger cone. I drew this picture to help:



To determine the radius of the larger cone, I will add $0.3 + 0.1 = 0.4$. The radius of the larger cone is 0.4 units.

$$0.4^2 + b^2 = 0.5^2$$
$$0.16 + b^2 = 0.25$$
$$b^2 = 0.09$$
$$b = 0.3.$$

The part of the radius of the bottom base found by the Pythagorean theorem is 0.3 units. When we add the length of the upper radius (because if you translate along the height of the truncated cone, it is equal to the remaining part of the lower base), the radius of the lower base is 0.4 units.

Let x units represent the height of the small cone.

Now that I have all the information I need, the problem is the same as Problem 1.

$$\frac{0.1}{0.4} = \frac{x}{x+0.4}$$
$$0.1(x+0.4) = 0.4x$$
$$0.1x + 0.04 = 0.4x$$
$$0.04 = 0.3x$$
$$\frac{0.04}{0.3} = x$$
$$\frac{4}{30} = \frac{2}{15} = x$$

The height of the large cone is $\frac{2}{15} + 0.4$ which is equal to $\frac{8}{15}$.

Volume of the small cone:

$$V = \frac{1}{3}\pi(0.1)^2\left(\frac{2}{15}\right)$$
$$= \left(\frac{1}{3}\right)\left(\frac{1}{100}\right)\left(\frac{2}{15}\right)\pi$$
$$= \frac{2}{4500}\pi$$
$$= \frac{1}{2250}\pi$$

Volume of the large cone:

$$V = \frac{1}{3}\pi(0.4)^2\left(\frac{8}{15}\right)$$
$$= \left(\frac{1}{3}\right)\left(\frac{16}{100}\right)\left(\frac{8}{15}\right)\pi$$
$$= \frac{128}{4500}\pi$$
$$= \frac{64}{2250}\pi$$

Volume of the truncated cone:

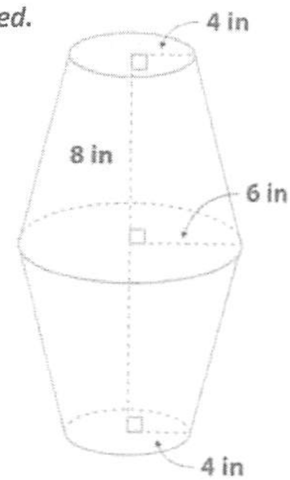
$$V = \frac{64}{2250}\pi - \frac{1}{2250}\pi$$
$$= \left(\frac{64}{2250} - \frac{1}{2250}\right)\pi$$
$$= \frac{63}{2250}\pi$$

The volume of the truncated cone is $\frac{63}{2250}\pi$ cubic units.

G8-M7-Lesson 21: Volume of Composite Solids

1. What volume of gel is required to completely fill up the lava lamp shown below? Note: 8 in. is the height of the truncated cone, not the lateral length of the cone.

Let x in. represent the height of the portion of the cone that has been removed.



I need to find the volume of the large cone and the volume of the cone that was removed like I did in the previous lesson.

$$\frac{4}{6}=\frac{x}{x+8}$$
$$4(x+8)=6x$$
$$4x+32=6x$$
$$32=2x$$
$$\frac{32}{2}=x$$
$$16=x$$

Volume of the removed cone:

$$V=\frac{1}{3}\pi(4)^2(16)$$
$$=\frac{256}{3}\pi$$

Volume of the cone:

$$V=\frac{1}{3}\pi(6)^2(24)$$
$$=\frac{864}{3}\pi$$

Volume of one truncated cone:

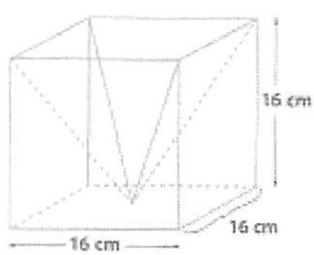
$$\frac{864}{3}\pi-\frac{256}{3}\pi=\left(\frac{864}{3}-\frac{256}{3}\right)\pi$$
$$=\frac{608}{3}\pi$$

The volume of gel needed to fill the lava lamp is $\frac{1216}{3}\pi\ \text{in}^3$.

There are two congruent truncated cones that make up the volume of the lava lamp. I need to multiply my result by 2 for the total volume.

2.

a. Write an expression for finding the volume of the prism with the pyramid portion removed. Explain what each part of your expression represents.



$$(16)^3-\frac{1}{3}(16)^3$$

I will subtract the volume of the pyramid from the volume of the cube.

The expression $(16)^3$ is the volume of the cube, and $\frac{1}{3}(16)^3$ is the volume of the pyramid. Since the pyramid's volume is being removed from the cube, we subtract the volume of the pyramid from the cube.

b. What is the volume of the prism shown above with the pyramid portion removed?

Volume of the prism:

$$V = (16)^3 = 4096$$

Volume of the pyramid:

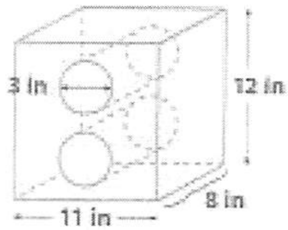
$$V = \frac{1}{3}(4096) = \frac{4096}{3}$$

I determined the volume by $4096 - \left(\frac{4096}{3}\right) = \frac{8192}{3}$.

The volume of the prism with the pyramid removed is $\frac{8192}{3}$ cm^3.

3. What is the approximate volume of the rectangular prism with two congruent cylindrical holes shown below? Use 3.14 for π. Round your answer to the tenths place.

The diagram shows the diameters of the cylindrical holes. The radius will be half the diameter. The height of the cylinder is the same as the width of the prism.



Volume of the prism:

$$V = (11)(8)(12) = 1056$$

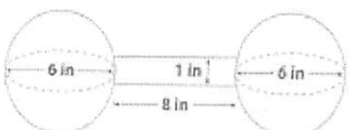
Volume of one cylinder:

$$V = \pi(1.5)^2(8) = 18\pi \approx 56.52$$

$$1056 - 56.52 - 56.52 = 942.96$$

The volume of the prism with the cylindrical holes is approximately 943 in^3.

4. What is the exact total volume of the barbell shown below? Note: The two spheres are joined together by a cylinder.



The diagram shows the diameters of the spheres. The radius will be half the diameter.

Volume of each sphere:

$$V = \frac{4}{3}\pi(3)^3 = 36\pi$$

Volume of cylinder:

$$V = (0.5)^2\pi(8) = 2\pi$$

$$36\pi + 36\pi + 2\pi = (36 + 36 + 2)\pi = 74\pi$$

The total volume of the barbell is 74π in^3.

G8-M7-Lesson 22: Average Rate of Change

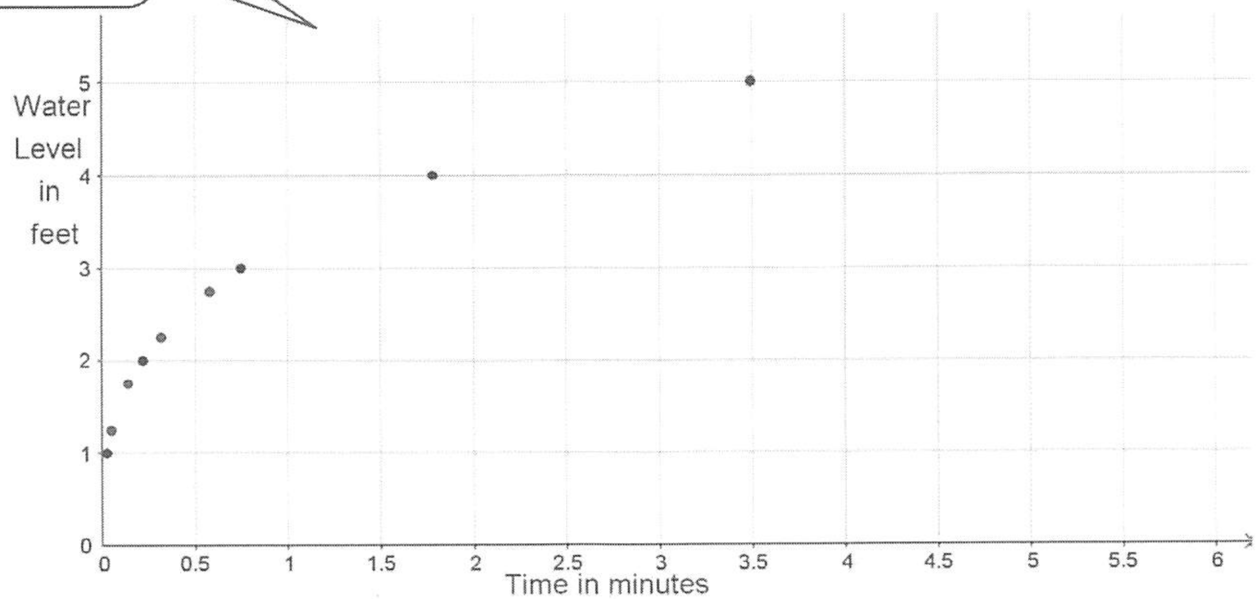
1. Complete the table below for more intervals of water levels of the cone discussed in class. Then, graph the data on a coordinate plane.

Time (in minutes)	Water Level (in feet)
0.028	1
0.055	1.25
0.15	1.75
0.22	2
0.32	2.25
0.58	2.75
0.75	3
1.78	4
3.49	5

I already know some of the times it takes to fill the cone from the work I did in class.

I will use the proportion $\frac{3}{\text{radius}} = \frac{7.5}{\text{water level}}$ to find the radius like we did in class. Next, I will determine the volume of the cone with the radius I just determined. I will divide this amount by the rate, 6 ft^3 per minute, at which the cone is being filled to give me the time it takes to fill the cone to the given heights.

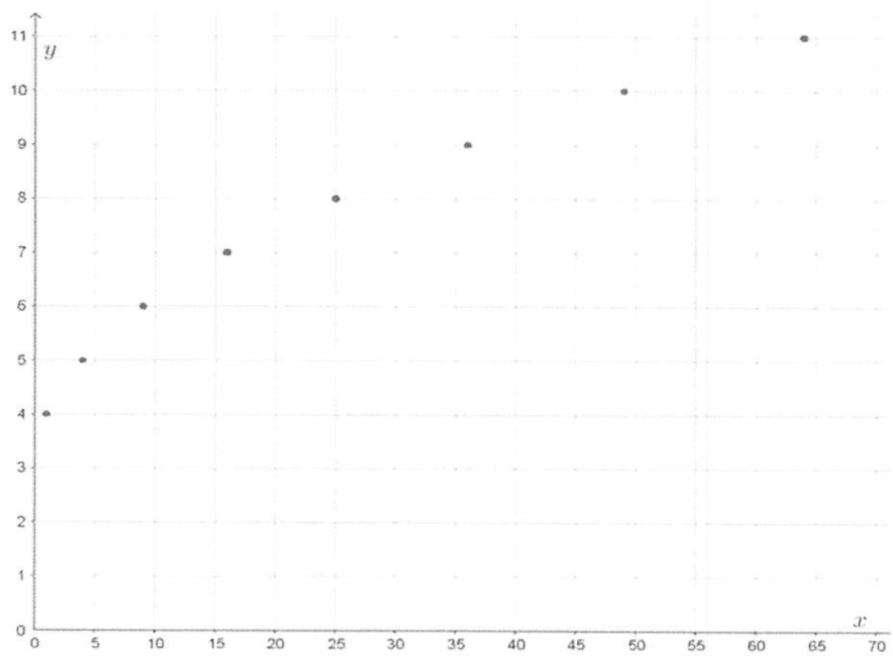
I will add to the graph we started in class.



2. Complete the table below, and graph the data on a coordinate plane. Compare the graphs from Problems 1 and 2. What do you notice? If you could write a rule to describe the function of the rate of change of the water level of the cone, what might the rule include?

The inputs are all perfect squares. I need to add 3 to the result of $\sqrt{x}$ for each of the outputs.

x	$\sqrt{x}+3$
1	4
4	5
9	6
16	7
25	8
36	9
49	10
64	11



The graphs are similar in shape. The rule that describes the function for the rate of change likely includes a square root. Since the graphs of functions are the graphs of certain equations where their inputs and outputs are points on a coordinate plane, it makes sense that the rule producing such a curve would be a graph of some kind of square root.

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G8-M7-Lesson 23: Nonlinear Motion

1. Suppose a ladder is 12 feet long, and the top of the ladder is sliding down the wall at a rate of 0.9 ft. per second. Compute the average rate of change in the position of the bottom of the ladder over the intervals of time from 0 to 0.5 seconds, 3 to 3.5 seconds, 7 to 7.5 seconds, 9.5 to 10 seconds, and 12 to 12.5 seconds. How do you interpret these numbers?

I can use a calculator to help with the substitution of the input, t, into the equation to get the output, d.

Input t	Output $d = \sqrt{144 - (12 - 0.9t)^2}$
0	$\sqrt{0} = 0$
0.5	$\sqrt{10.60} \approx 3.26$
3	$\sqrt{57.51} \approx 7.58$
3.5	$\sqrt{65.68} \approx 8.1$
7	$\sqrt{111.51} \approx 10.56$
7.5	$\sqrt{116.44} \approx 10.79$
9.5	$\sqrt{132.1} \approx 11.49$
10	$\sqrt{135} \approx 11.62$
12	$\sqrt{142.56} \approx 11.94$
12.5	$\sqrt{143.44} \approx 11.98$

We did an example like this in class where d represented the distance from the bottom of the ladder to the corner where the wall intersected the floor.

The average rate of change between 0 *and* 0.5 *seconds is*

$$\frac{3.26 - 0}{0.5 - 0} = \frac{3.26}{0.5} = 6.52.$$

The average rate of change between 3 *and* 3.5 *seconds is*

$$\frac{8.1 - 7.58}{3.5 - 3} = \frac{0.52}{0.5} = 1.04.$$

The average rate of change between 7 *and* 7.5 *seconds is*

$$\frac{10.79 - 10.56}{7.5 - 7} = \frac{0.23}{0.5} = 0.46.$$

The average rates are not the same over the time intervals. This means that the ladder is not sliding down the wall at a constant rate. This means the average rate of change is not linear.

The average rate of change between 9.5 *and* 10 *seconds is*

$$\frac{11.62-11.49}{10-9.5}=\frac{0.13}{0.5}=0.26.$$

The average rate of change between 12 *and* 12.5 *seconds is*

$$\frac{11.98-11.94}{12.5-12}=\frac{0.04}{0.5}=0.08.$$

The average rates are getting smaller as the ladder slides down the wall.

The average speeds show that the rate of change in the position of the bottom of the ladder is not linear. Furthermore, it shows that the rate of change in the position at the bottom of the ladder is quick at first, 6.52 *feet per second in the first half second of motion, and then slows down to* 0.08 *feet per second in the half second interval from* 12 *to* 12.5 *seconds.*

2. Will any length of ladder, L, and any constant speed of sliding of the top of the ladder v ft. per second, ever produce a constant rate of change in the position of the bottom of the ladder? Explain.

No, the rate of change in the position at the bottom of the ladder will never be constant. We showed that if the rate were constant, there would be movement in the last second of the ladder sliding down that wall that would place the ladder in an impossible location. That is, if the rate of change were constant, then the bottom of the ladder would be in a location that exceeds the length of the ladder. Also, we determined that the distance that the bottom of the ladder is from the wall over any time period can be found using the formula $d=\sqrt{L^2-(L-vt)^2}$*, which is a nonlinear equation. Since graphs of functions are equal to the graph of a certain equation, the graph of the function represented by the formula* $d=\sqrt{L^2-(L-vt)^2}$ *is not a line, and the rate of change in position at the bottom of the ladder is not constant.*

I tried this experiment with a book, and I could see that the book slid down the wall quickly at first but slowed down when it hit the floor.

In class, we learned that the equation $d=\sqrt{L^2-(L-vt)^2}$ is not linear. This means the average rates of change will not be constant.