Homework Helpers

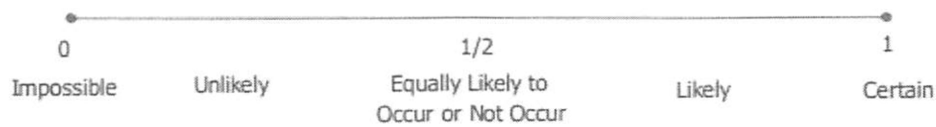
Grade 7
Module 5

G7-M5-Lesson 1: Chance Experiments

Probability Scale

The probability of an event will fall somewhere on the probability scale.

Probability Scale



Decide whether each event is impossible, unlikely, equally likely, likely, or certain to occur.

1. It will start raining gum drops on the way home from school.

   Impossible

2. An even number will be chosen from a bag containing items numbered 1 through 20.

   Equally Likely

   > I know there are 20 numbers in the bag, and 10 of them are even. Therefore, the probability will be $\frac{10}{20} = \frac{1}{2}$.

3. I will roll a composite number on a six-sided number cube with sides numbered 1 through 6.

   Unlikely

   > There are two composite numbers (4 and 6) on a six-sided number cube. That means two out of the six possible numbers are composite, making the probability less than half.

4. A letter chosen from the alphabet is a consonant.

   Likely

   > Twenty-one of the twenty-six letters in the alphabet are consonants, which means the probability is greater than half.

G7-M5-Lesson 1: Chance Experiments

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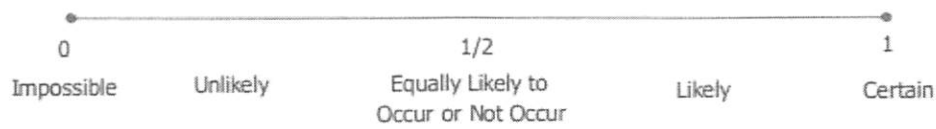

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4. A letter chosen from the alphabet is a consonant.

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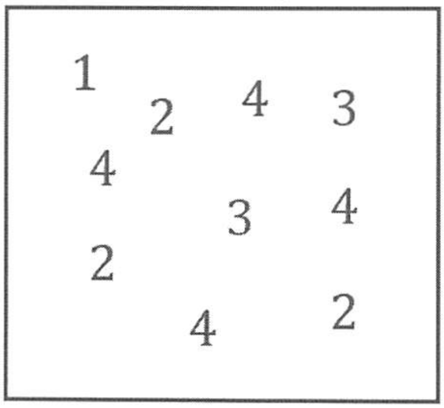
   > Twenty-one of the twenty-six letters in the alphabet are consonants, which means the probability is greater than half.

5. A number will be randomly drawn from the box shown below. Decide where each event would be located on the probability scale. Then, place the letter for each event on the appropriate location on the probability scale.

There are 10 possible outcomes.



Three appears twice, which means the event is unlikely. However, there are more possible outcomes for event E than for event B, which means event E should be to the right of event B, but event E is still unlikely.

Event:

Seven of the numbers are even.

A. An even number is drawn.

B. A 1 is drawn.

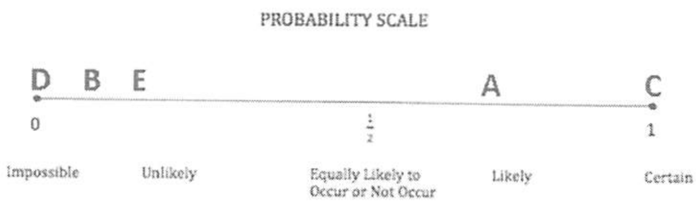
All the outcomes are numbers.

C. A number is drawn.

There are no letters.

D. A letter is drawn.

E. A 3 is drawn.



G7-M5-Lesson 2: Estimating Probabilities by Collecting Data

Cole is eating candy from a bag that consists of different-colored pieces. Cole randomly picked pieces from the bag and recorded the number of each color in the table below.

Color	Number
Red	4
Brown	7
Green	4
Yellow	10
Blue	7
Orange	8

a. How many pieces of candy are in the bag?

40

> I add the number of each color of candy together to find the total amount of candy.

b. How many pieces of candy are yellow?

10

If Cole randomly selected a piece of candy from the bag:

c. What is the estimated probability of Cole eating an orange piece of candy?

$\frac{8}{40} = \frac{1}{5} = 20\%$

> The number of observed occurrences is 8, and the total number of observations is 40.

d. What is the estimated probability Cole will eat either a brown or blue piece of candy?

$\frac{14}{40} = \frac{7}{20} = 35\%$

> The number of observed occurrences is 14 because there are 7 brown pieces of candy and 7 blue pieces of candy in the bag.

e. If the bag of candy has 600 pieces of candy, how many pieces would you expect to be red?

$\frac{4}{40} = \frac{1}{10} = 10\%$

> I find the estimated probability of choosing a red piece of candy. I can use this percentage to estimate the total number of red pieces of candy in the bag.

An estimate for the number of red pieces of candy would be 60 because 10% of 600 is 60.

G7-M5-Lesson 3: Chance Experiments with Equally Likely Outcomes

The sample space is all the possible outcomes.

For each of the following chance experiments, list the sample space.

1. Selecting a marble from a bag of 6 green marbles, 8 yellow marbles, and 4 red marbles

   Green, yellow, and red

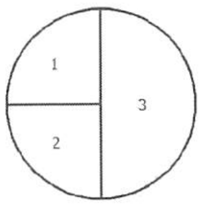
When listing the sample space, I list all the letters in the word *homework*. However, the letter *o* only needs to be listed once.

2. Selecting a letter from the word *homework*

   h, o, m, e, w, r, and k

3. Spinning the spinner below

   1, 2, and 3



It does not matter if the outcomes are equally likely; the sample space just focuses on the possible outcomes.

For each of the following problems, decide if the two outcomes listed are equally likely to occur. Give a reason for your answer.

The letters i and b both occur twice in the word *probability*.

4. Selecting the letters i or b from the word *probability*

   Yes, both i and b occur the same number of times in the word probability.

5. Selecting a red or blue uniform shirt when Lincoln has 4 red uniform shirts and 5 blue uniform shirts

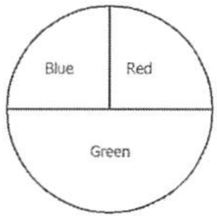
   No, Lincoln has a slightly higher chance of picking a blue shirt.

Lincoln has a different number of each color shirt, which means each outcome is not equally likely.

6. Landing on blue or green on the spinner below

   No, it is more likely to land on green than on blue.

The area of green on the spinner is larger than the area of blue. Therefore, the outcomes are not equally likely.



G7-M5-Lesson 4: Calculating Probabilities for Chance Experiments with Equally Likely Outcomes

Calculate the Probability of Events when Outcomes are Not Equally Likely

1. In a middle school orchestra, there are 6 sixth graders, 11 seventh graders, and 8 eighth graders.

   There are 25 students in the orchestra because $6 + 11 + 8 = 25$. I know this represents the number of possible outcomes.

   a. If one student is randomly chosen to complete a solo, what is the probability that a seventh grader is chosen?

   $\frac{11}{25}$

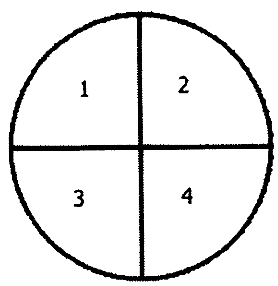
   The number of favorable outcomes is the number of seventh graders in the orchestra.

   b. If one student is randomly chosen, is it equally likely to pick a sixth, seventh, or eighth grader? Explain.

   No, there are not the same number of sixth, seventh, or eighth graders.

   In order for outcomes to be equally likely, the number of favorable outcomes must be the same.

Calculate the Probability of Events with Equally Likely Outcomes



2. Use the spinner to the right to answer the following questions.

   a. What is the probability of landing on an even number?

   $\frac{2}{4} = \frac{1}{2}$

   There are 4 possible outcomes because there are 4 sections on the spinner.

   There are two even numbers (2 and 4) on the spinner.

b. What is the probability of landing on a composite number?

$\frac{1}{4}$

There is only one composite number, 4, on the spinner.

c. Is landing on each section of the spinner equally likely to occur? Explain.

Yes, each section has the same area.

3. A chance experiment consists of rolling a number cube with the numbers 1–6 on the faces of the cube and flipping a coin.

a. List the sample space of this chance experiment. List the probability of each outcome in the sample space.

Sample Space: $1t, 1h, 2t, 2h, 3t, 3h, 4t, 4h, 5t, 5h, 6t, 6h$

The probability of each outcome is $\frac{1}{12}$.

There are 12 possible outcomes, and they are all equally likely to occur.

b. What is the probability of getting the number 5 on the number cube and a tails on the coin?

$\frac{1}{12}$

It is estimated that this outcome will occur once in 12 trials.

c. What is the probability of getting a heads on the coin and an odd number on the number cube?

$\frac{3}{12}$, *or* $\frac{1}{4}$

It is estimated that this outcome will occur three times in 12 trials.

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G7-M5-Lesson 5: Chance Experiments with Outcomes That Are Not Equally Likely

1. Charlie is training for a race and is supposed to run every day during the week. The table below shows the estimated probabilities of running 1, 2, 3, 4, 5, 6, or 7 days a week.

Number of Days	1	2	3	4	5	6	7
Probability	0.1	0.1	0.1	0.2	0.3	0.2	0

Find the probability that Charlie will

> In order to find the probability of an event, I add the probability of each of the desired outcomes together.

a. Run more than 4 days during the week.

> "More than 4 days" means that Charlie will run 5, 6, or 7 days.

$0.3 + 0.2 + 0 = 0.5$

The probability that Charlie will run more than 4 *days a week is* 0.5.

b. Run at most 3 days.

> "At most 3 days" means that Charlie will run 3 or fewer days.

$0.1 + 0.1 + 0.1 = 0.3$

The probability that Charlie will run at most 3 *days is* 0.3.

c. Not run exactly 1 day.

> I can solve this problem two ways:
> 1. I can find the sum of all the probabilities, except for the probability of running 1 day.
> 2. The total probability is 1, so I can subtract $1 - P(1)$.

Method 1:

$0.1 + 0.1 + 0.2 + 0.3 + 0.2 + 0 = 0.9$

Method 2:

$1 - 0.1 = 0.9$

The probability that Charlie will not run exactly 1 *day is* 0.9.

2. Sarah surveyed her friends to determine the number of pets each friend has. The survey results are shown in the table below.

Number of Pets	0	1	2	3	4
Number of Friends	8	6	3	1	2

a. How many friends did Sarah survey?

$8 + 6 + 3 + 1 + 2 = 20$

Sarah surveyed 20 friends.

b. What is the probability that a randomly selected friend does not have any pets? Write your answer as a fraction in lowest terms.

$\frac{8}{20} = \frac{2}{5}$

The 8 represents the number of friends who have zero pets, and the 20 represents the total number of friends surveyed.

The probability that Sarah will select a friend who does not have any pets is $\frac{2}{5}$.

c. The table below shows the possible number of pets and the probabilities of each number of pets. Complete the table by writing the probabilities as fractions in lowest terms.

Number of Pets	0	1	2	3	4
Probability	$\frac{8}{20} = \frac{2}{5}$	$\frac{6}{20} = \frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$	$\frac{2}{20} = \frac{1}{10}$

To find each probability, I put the number of favorable outcomes in the numerator and the total number of outcomes in the denominator. If possible, I simplify the fraction.

d. Writing your answers as fractions in lowest terms, find the probability that the student:

i. Has fewer than 3 pets.

$$\frac{2}{5}+\frac{3}{10}+\frac{3}{20}$$

$$\frac{8}{20}+\frac{6}{20}+\frac{3}{20}$$

$$\frac{17}{20}$$

In order to add the probabilities of a friend having 0, 1, or 2 pets, I find a common denominator or use the fractions that are not simplified from the table.

ii. Does not have exactly 4 pets.

$$1-\frac{1}{10}$$

$$\frac{10}{10}-\frac{1}{10}$$

$$\frac{9}{10}$$

The total probability is 1, so I would subtract the probability of a friend having four pets from 1.

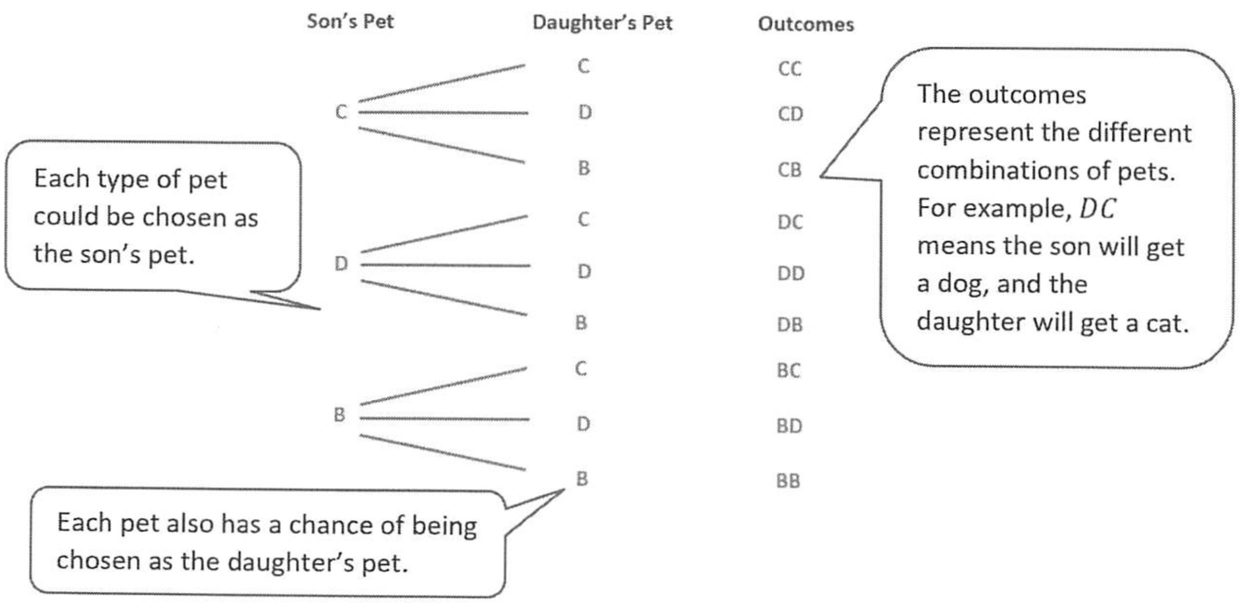
$\frac{10}{10}$ represents the same value as 1.

NOTE: I could also add the probabilities of a friend having 0, 1, 2, or 3 pets together to get the probability of not having 4 pets.

G7-M5-Lesson 6: Using Tree Diagrams to Represent a Sample Space and to Calculate Probabilities

1. The Johnson family has decided to get two new pets, one for their son and one for their daughter. Mr. and Mrs. Johnson are allowing the kids to choose between a cat, a dog, or a bird. Each type of pet has an equally likely chance of being chosen.

   a. Using C for cat, D for dog, and B for bird, develop a tree diagram that shows the nine possible outcomes for the two different types of pets.



   b. What is the probability of the son choosing a bird and the daughter choosing a cat?

$\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

Since each pet has an equal chance to be chosen, the probability for each type of pet is $\frac{1}{3}$.

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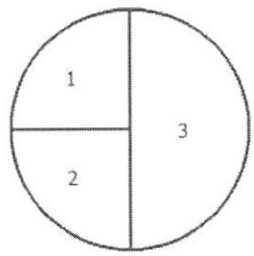
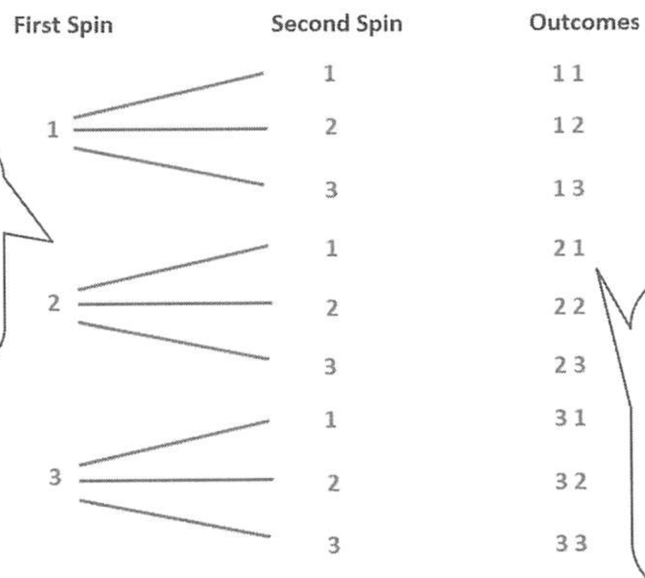
c. Is the probability that both children will choose a dog the same as the probability that both children will choose a bird? Explain.

The probability of both children choosing a dog is $\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$*. The probability of both children choosing a bird is also* $\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$*. Therefore, the probability of both children choosing a dog is the same as the probability of both children choosing a bird.*

2. Ms. Bailey's class is playing a game where they have to spin the spinner below.



a. Develop a tree diagram showing the nine possible outcomes of spinning the spinner twice.



At this point, I just list the sample space and don't worry about the probability of each outcome.

The outcomes represent the different combinations of the two spins but does not yet indicate the probability of each outcome.

b. What is the probability that a student will spin a 2 on the first spin and a 3 on the second spin?

$(0.25)(0.5) = 0.125$

The probability of spinning a 2 is 0.25 because this section covers 25% of the spinner's area. The probability of spinning a 3 is 0.5 because this section covers 50% of the spinner's area.

c. What is the probability that the spinner will land on the 1 for both spins?

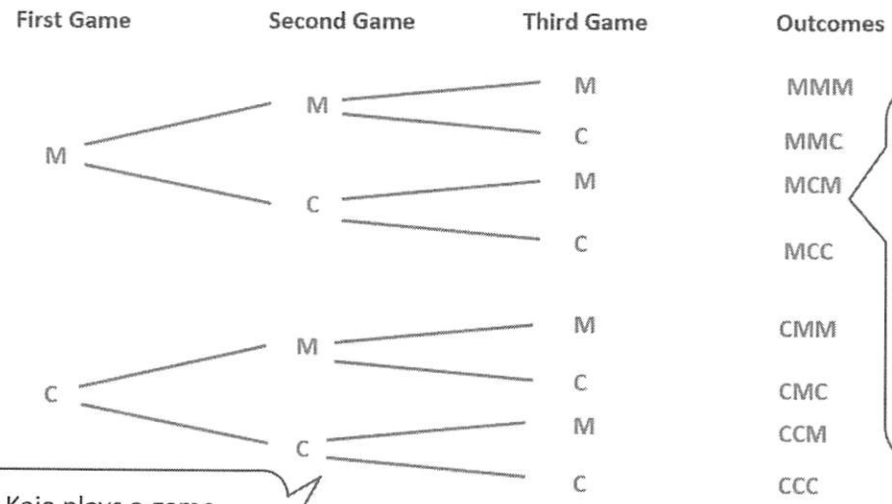
$(0.25)(0.25) = 0.0625$

The probability of spinning a 1 on either spin is 0.25 because this section covers 25% of the spinner's area.

G7-M5-Lesson 7: Calculating Probabilities of Compound Events

Kaia's four kids are arguing over the type of game they want to play. Therefore, Kaia agrees to roll a four-sided die with sides numbered 1, 2, 3, and 4. If the die lands on a 1, 2, or 3, they will play a matching game, and if the die lands on a 4, they will play a card game.

a. Kaia agrees to play three games with her kids. Create a tree diagram to show the different types of games they may play.



Although these are the 8 possible outcomes, I know they are not equally likely to occur.

Each time Kaia plays a game, she has a chance of playing a matching game or card game.

b. What is the probability that all three games will be a matching game?

$P(MMM) = (0.75)(0.75)(0.75) = 0.421875$

The probability that each game played will be a matching game is 0.75 because 3 of the 4 sides of the die represent a matching game.

The probability that a card game will be played is 0.25 because 1 of the 4 sides of the die represents a card game.

c. What is the probability that at least two card games will be played?

The possible outcomes and their probabilities are

$P(MCC) = (0.75)(0.25)(0.25) = 0.046875$

$P(CMC) = (0.25)(0.75)(0.25) = 0.046875$

$P(CCM) = (0.25)(0.25)(0.75) = 0.046875$

$P(CCC) = (0.25)(0.25)(0.25) = 0.015625$

In order to find the total probability of playing at least two card games, I calculate the sum of these probabilities.

The probability of playing at least two card games is

$0.046875 + 0.046875 + 0.046875 + 0.015625 = 0.15625$

d. What is the probability that Kaia and her kids will play at least one matching game?

$$P(CCC) = (0.25)(0.25)(0.25) = 0.015625$$

The only time the family will not play at least one matching game is when they play three card games.

$P(no\ M) = 1 - P(CCC)$

$P(no\ M) = 1 - 0.015625$

$P(no\ M) = 0.984375$

The probability of playing at least one matching game is the total probability (which is 1) minus the probability of playing three card games.

G7-M5-Lesson 8: The Difference Between Theoretical Probabilities and Estimated Probabilities

Predicting Theoretical Probability

1. Consider the data you collected in class when you taped 10 pennies in a tall stack. Would the probability of landing on a head be more likely or less likely if we taped more than 10 pennies in a stack? Explain.

   As we create a bigger stack, the probability of landing on a head becomes less likely because the larger stacks will land on their sides more often.

   I can perform the experiment if I am not sure how to answer this question.

   Large stacks of pennies are difficult to balance standing upright, so they will not land standing upright very often.

2. If you created a stack of pennies shorter than 10 pennies, how would the probability of landing on a head change?

   A smaller stack of pennies would increase the probablity of landing on a head because it is easier to stand small stacks upright.

   As the stack of pennies gets smaller, the probability of landing on a head increases.

Estimated Probability

3. Assume we taped 3 pennies into a stack, tossed the stack 20 times, and recorded our results in the table below.

Number of Tosses	Total Number of Heads so Far	Relative Frequency of Heads so Far (to the nearest hundredth)
1	0	$\frac{0}{1} = 0.0$
5	2	$\frac{2}{5} = 0.4$
10	3	$\frac{3}{10} = 0.3$
15	5	$\frac{5}{15} = 0.\overline{3}$
20	6	$\frac{6}{20} = 0.3$

In order to calculate the relative frequency, I set up a fraction to show the number of heads out of the total number of tosses.

a. Complete the table by calculating the relative frequencies. If necessary, round to the nearest hundredth.

b. What is your estimated probability that our stack of pennies will land heads up when tossed? Explain.

My estimated probability that our stack of pennies will land heads up is 0.3 ***because all of the relative frequencies are close to this number.***

Answers will vary but should be based on the relative frequencies.

G7-M5-Lesson 9: Comparing Estimated Probabilities to Probabilities Predicted by a Model

Picking Green! This is a game similar to the one you played in class, where you try to pick as many green chips as possible before picking one white chip. One bag has the same number of green and white chips, and the ratio of green to white chips in the second bag is unknown.

After experimenting, I either choose Bag A or Bag B in hopes of picking the most green chips while avoiding white chips.

Laura and Carly completed an experiment, and their results are shown in the tables below.

Laura's Results:

Bag	Number of Green Chips Picked	Number of White Chips Picked
A	35	15
B	22	28

Carly's Results:

Bag	Number of Green Chips Picked	Number of White Chips Picked
A	7	8
B	9	6

1. If all you know about the bags are the results from Laura's research, which bag would you select for the game? Explain.

   I would choose Bag A because Laura's results show that she picked a lot more green chips than white chips from Bag A.

   In order to win, I want to pick the bag that I think has the most green chips.

2. If all you know about the bags are the results from Carly's research, which bag would you select for the game? Explain.

   I would choose Bag B because Carly's results show that she picked a few more green chips than white chips from Bag B.

   Both Bag A and Bag B had similar results, but Carly picked a few more green chips from Bag B.

3. Whose research gives you a better indication of the makeup of green and white chips in each bag? Explain.

Laura's results would be a better indication of the makeup of green and white chips in each bag because she collected more data than Carly.

The more data we collect, the closer the outcome is to the theoretical probability.

I know that the more outcomes carried out, the closer the relative frequency is to the theoretical probability.

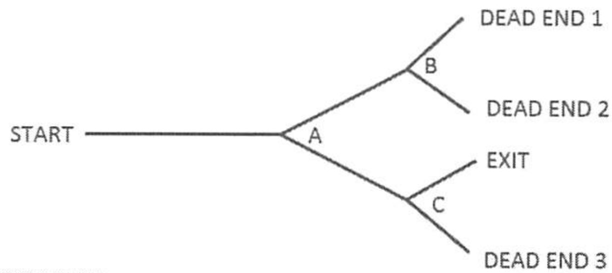
4. If there were three colors of chips in each bag, how would you collect data in order to choose a bag?

My data collection would be the same. I would just have to extend my table to include a third color.

G7-M5-Lesson 10: Conducting a Simulation to Estimate the Probability of an Event

Predicting a Mouse's Path

1. Samantha bought her hamster a new maze for his cage, which is shown below. The hamster can only exit the maze at one point. At each point where the hamster has to decide which direction to go, assume that it is equally likely to go in either direction. At each decision point, A, B, and C, it must decide whether to go left (L) or right (R).



> If needed, I can create a tree diagram to determine the sample space.

a. List the possible paths of a sample space for the paths the hamster can take. For example, if the hamster goes left at decision point A and then right at decision point B, then the path would be denoted LR.

The sample space is LL, LR, RL, RR.

b. Are the paths in the sample space equally likely? Explain.

At each decision point there are two choices, which are equally likely. Therefore, each path in the sample space is equally likely as the other paths.

c. What is the theoretical probability of the hamster finding the exit?

> The only path that leads to the exit is RL.

Only one of the four possible paths will lead the hamster to the exit. Therefore, the probability of the hamster reaching the exit is $\frac{1}{4}$.

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d. What is the theoretical probability of the hamster reaching a dead end?

Three of the four possible paths will lead to a dead end, which means the probability is $\frac{3}{4}$.

e. Based on the set of simulated paths, estimate the probabilities that the hamster arrives at the exit and any of the dead ends. Explain.

There are 30 trials listed here.

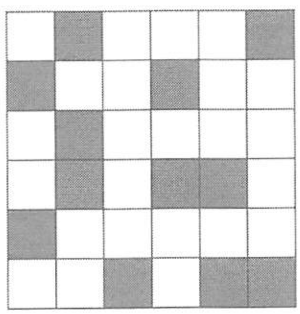
RR	RL	LR	LL	LR	LL	RR	LL	RR	RL
LL	LR	LR	RL	RR	RL	LR	RL	RL	LL
RL	LR	LL	RR	RL	RL	LR	RR	LL	RR

The probability of the hamster reaching the exit is $\frac{9}{30}$ *because* 9 *of the outcomes resulted in the hamster going RL, which is the only path that leads to the exit.*

The probability of the hamster reaching a dead end is $1-\frac{9}{30}=\frac{21}{30}$ *because all the other paths lead toward a dead end.*

Using Simulation with a Dart Board

2. Suppose a dart board is made up of the 6×6 grid of squares shown below. Also, suppose that when a dart is thrown, it is equally likely to land on any one of the 36 squares. A point is won if the dart lands on one of the 12 red squares. Zero points are earned if the dart lands in a white square.



a. For one throw of a dart, what is the probability of winning a point?

$\frac{12}{36}=\frac{1}{3}$

I win a point if I land on one of the 12 red squares. I also know there are 36 total squares.

b. Dylan wants to use a six-sided number cube with the sides numbered 1 through 6 to simulate the results of one dart. How could he assign the six numbers on the number cube to create an appropriate simulation?

Dylan can assign 1 and 2 to simulate winning a point and the numbers 3, 4, 5, and 6 to simulate not winning a point.

> Since there are six different numbers on a number cube, I know that 2 of the numbers must represent winning a point because $\frac{2}{6} = \frac{1}{3}$. The other four numbers will represent not winning a point.

Suppose a game consists of throwing four darts. A trial consists of four rolls of the number cube. Based on your suggestion in part (b) and the following simulated rolls,

1234	3321	5624	2451	6253
1625	3452	6115	[2511]	2361
(5436)	[2251]	5461	1253	(6344)
5513	(6634)	[5112]	3426	(5534)

c. What is the probability that none of the four darts will score a point?

Note: These four trials are circled above.

$\frac{4}{20} = \frac{1}{5}$

> I determine how many of the 20 trials do not include a 1 or 2 because these numbers represent winning a point.

d. What is the probability that three of the darts will score a point?

Note: These three trials are boxed above.

$\frac{3}{20}$

> I determine how many of the 20 trials have three rolls that are either a 1 or 2.

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G7-M5-Lesson 11: Conducting a Simulation to Estimate the Probability of an Event

1. George typically takes 6 free throws during one basketball game. Usually, George makes 80% of his free throw shots. Design a simulation to estimate the probability that George will make at least 4 free throws during his next game.

   a. How would you simulate the number of free throws George makes and misses?

   I could put 10 slips of paper in a bag, 8 of them labeled M, for a made shot, and 2 of them labeled L, for a missed shot.

   I can do this in more than one way but would have to have either 5 or 10 possible outcomes since 80% represents $\frac{8}{10}$ or $\frac{4}{5}$.

   b. What constitutes a trial for this simulation?

   A trial for this simulation would be picking 6 pieces of paper from the bag, replacing each piece of paper before picking again.

   Each piece of paper represents one shot.

   c. What constitutes a success in a trial in this problem?

   A success would be looking at the 6 pieces of paper that were picked and seeing 4 or more papers that say "made."

   A success is a simulation where the results show George making 4 or more free throws.

   d. Carry out 12 trials, list your results, and compute an estimate of the probability that George will make at least 4 of his free throw shots.

   I will refer back to part (b) to remember what a trial consists of and repeat this 12 times.

MMMLML	*LMLLMM*	*MMLMMM*
LMLLMM	*LMMMMM*	*LMLMMM*
MLLMMM	*MMLMLL*	*MMMMMM*
MMLMMM	*LLMLMM*	*MMMLMM*

8 of the 12 trials show at least four made (M) shots.

$$\frac{8}{12} = \frac{2}{3}$$

G7-M5-Lesson 12: Applying Probability to Make Informed Decisions

A recall has been issued for one type of small toy car and one type of large toy car because so many of the cars have small pieces breaking off.

Brett believed that the probability of having a large toy car that was recalled is bigger than the probability of having a small toy car that was recalled because the large toy car would have more parts that could break. However, Rachel believed that the probability of having a recalled large toy car is the same as having a recalled small toy car.

a. Simulate inspecting a small toy car by pulling a single card from a standard 52-card deck of cards. Let a heart simulate a recalled small toy car and all other cards simulate a safe small toy car. Do 50 trials, and compute an estimate of the probability that a small toy car is recalled.

> In order to have an accurate estimated probability, I replace each card before picking a new one.

Students pick 50 cards (replacing them each time) and record the results. Students then identify the number of hearts chosen out of the 50 trials. Since hearts represent $\frac{1}{4}$ of a standard 52-card deck, the estimated probability should be close to $\frac{1}{4}$.

b. Simulate inspecting a large toy car by pulling a single card from a standard 52-card deck of cards. Let a black face card simulate a recalled large toy car and all other cards simulate a safe large toy car. Do 50 trials, and compute an estimate of the probability that a large toy car is recalled.

> I know a face card is a Jack, Queen, or King.

Students again pick 50 cards (replacing them each time) and record the results. Students then identify the number of black face cards chosen out of the 50 trials. Since black face cards represent $\frac{6}{52}$ or $\frac{3}{26}$ of the deck of cards, the estimated probability should be close to $\frac{3}{26}$.

c. For this problem, suppose that the two simulations provide accurate estimates of the probability of a recalled small toy car and a recalled large toy car. Compare your two probability estimates, and decide whether Brett's or Rachel's belief was more reasonable than the other. Explain your reasoning.

Neither person had the correct idea. The probability of having a recalled small toy car was greater than the probability of having a recalled large toy car.

> Although the estimates will be different, there is a greater chance of picking a heart than a black face card from a deck of cards. Therefore, I know that it is more likely to have a greater probability in part (a) than in part (b).

G7-M5-Lesson 13: Populations, Samples, and Generalizing from a Sample to a Population

1. For each of the following questions: (1) Describe how you would collect data to answer the question, and (2) decide whether it would result in a sample statistic or a population characteristic.

   a. How many pets do people own in my class?

      (1) I can provide all students in my class a slip of paper and have them write the number of pets they have on the piece of paper.

      (2) The result would be a population characteristic because I would be gathering data from the entire population (my class).

      I am able to collect data from the entire population because the population size is small.

   b. How many pets do people own in my city?

      There are too many people in my city to collect data from everyone, so I only collect data from a sample.

      (1) I can stand outside a grocery store and ask customers as they enter the store how many pets they own.

      (2) The result would be a sample statistic because I would be gathering information from only a small subset of the entire population (my city).

2. Identify a question that would lead to collecting data from the given set as a population and one where the data could be a sample from a larger population.

   a. The entire seventh grade

      The entire seventh grade could be a population when determining the number of seventh grade student absences on a given day.

      Data that could be collected easily, maybe from a computer, can be collected from an entire population.

      The entire seventh grade could be a sample when determining the number of hours a middle school student sleeps on a school night.

      I would want to collect data that can be summarized from a sample to generalize to the population.

b. The entire school district

The entire school district might be a population when determining math scores on the state assessment for the district.

The entire school district might be a sample when determining the median family income in the state.

G7-M5-Lesson 14: Selecting a Sample

1. Would any of the following provide a random sample of the length of words in a children's book? If not, explain.
   a. Placing all the words in a bag and picking a sample of words from the bag

      This method would provide a random sample.

   b. Finding the length of the last word on each page of the book

      This would not be a random sample because the book may be a rhyming book, which would make the last words on each page similar to each other and maybe not a good representation of the other words in the book.

      If I use a method to find a sample, it eliminates the randomness and is not a random sample.

2. Indicate whether the following are random samples from the given population, and explain why or why not.
   a. Population: All families in the city; sample includes people sitting at one bus stop.

      No, because the sample only includes people in one part of the city and only people who ride the bus.

      This sample only includes a specific group of people from my city.

   b. Population: Teachers at a school; sample selected by putting names into a bowl and drawing the sample from the bowl.

      Yes, all the teachers have the same opportunity to be chosen.

3. What questions about the samples and populations might you want to ask if you saw the following headlines in a newspaper?
   a. Soccer is the new favorite sport of children!

      Answers may vary.

      How many people were interviewed? Was the survey conducted at a soccer field?

      I have to think of ways that a biased sample may have been chosen.

   b. Spicy Mint Gum is favored by 80% of consumers.

      What were the other choices? How many people were surveyed?

G7-M5-Lesson 15: Random Sampling

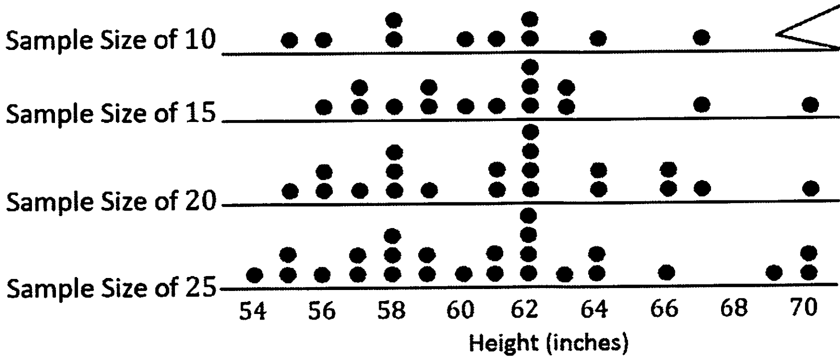
Consider the distribution below:

1. What would you expect the distribution of a random sample size of 10 from the population look like?

   I would expect that a majority of the 10 data values will be between 56 and 64, with maybe one data point smaller than 56 and two data points larger than 64.

> I know the sample should resemble the population, but there is more than one correct answer.

2. Random samples of different sizes that were selected from the population in Problem 1 are displayed below. How did your answer to Problem 1 compare to this sample of 10?



> There is one value smaller than 56 but only one data value greater than 64.

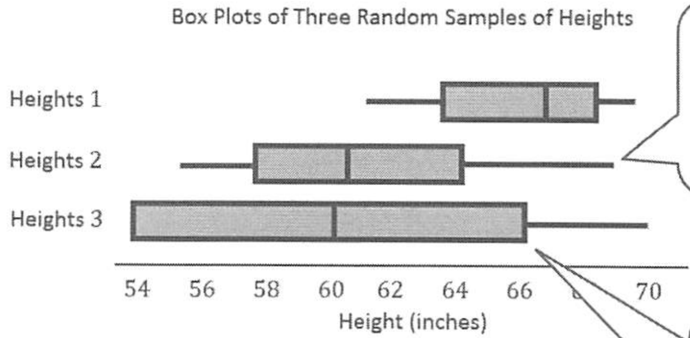
   My expectation from Problem 1 closely matches the sample given in the first dot plot here.

I notice that the dot plots are all similarly shaped. As the sample size gets larger, the dot plot looks even more like the dot plot of the population.

3. Why is it reasonable to think that these samples could have come from the above population?

Each of the dot plots has a similar shape to the dot plot that represents the population.

4. Which of the box plots could represent a random sample from the distribution? Explain your thinking.



I know the lines extending from the boxes extend to the smallest data value and the largest data value.

The box part of the box plot represents the middle half of the data. The line separating the box into two represents the median of the data.

The first sample is probably not from the distribution because the middle half of the data is too high for the data provided.

The second sample could be a sample of the distribution because the middle half of the data seems to represent the same heights as the middle half of the data in the original distribution.

The third sample is not from the distribution because the range for the middle half of the data is too big.

G7-M5-Lesson 16: Methods for Selecting a Random Sample

The population is all the teachers in my school.

1. The suggestions below describe ways to choose a random sample of teachers in your school that were made and vetoed. Explain why you think each was vetoed.

   a. Stand in the teachers' lounge, and use every teacher that enters before school.

   Teachers who do not enter the teachers' lounge in the morning do not have a chance to be selected, so the sample would not be random.

   In order for a sample to be random, everyone in the population must have an equal chance of being selected.

   b. Use all the science teachers in the school.

   The teachers who do not teach science do not have a chance to be selected, so the sample would not be random.

2. The school wanted random seventh graders to complete a survey every week in order to gain information about students' thoughts about the school.

   a. Describe how the school might choose a random sample of 25 seventh graders from the total of 120 seventh graders in the school.

   Each seventh grade student could be assigned a number, and then the numbers could be placed in a bag. Every week, 25 numbers could be pulled from the bag to have students complete the survey.

   I could also put all 120 names into a bag and pick 25 names every week.

   b. There are 40 weeks during the school year. If a random sample of 25 seventh graders is picked every week, would every student be chosen at least once? Why or why not?

   The probability of being chosen each week is $\frac{25}{120}$ or approximately 21%. Although the number of chances to get picked is high, the probability is low.

   I could carry out the investigation to determine if it is likely for every student to be chosen.

G7-M5-Lesson 17: Sampling Variability

1. Holly is trying to convince her mom that she needs a fancy dress for prom. She wants to estimate the mean price girls at her school pay (in dollars) for prom dresses at this time. Holly selects a random sample of 12 girls from her school and asks what they paid for their prom dress. The results are shown below.

$$100 \quad 54 \quad 32 \quad 97 \quad 68 \quad 89 \quad 142 \quad 61 \quad 77 \quad 106 \quad 96 \quad 49$$

   a. Holly will estimate the mean dress cost of all dresses bought by girls at her school by calculating the mean for her sample. Calculate the sample mean, and record your answer below.

$$\frac{100+54+32+97+68+89+142+61+77+106+96+49}{12} = \frac{971}{12} \approx 80.9$$

To calculate the mean, I add all the data values together and then divide by the number of data values, 12.

The mean cost of a prom dress is approximately $\$81.00$.

   b. If Holly collected another sample of dress prices, would the result be the same?

It is not likely that another sample would have the same mean as this one.

It is very rare that two samples will result in the exact same mean.

   c. Explain why the means of a variety of samples will be different.

Sample variability explains that there will be differences between samples of the same population, which would result in different means.

2. Think about the mean number of pets for all students at your school.

   a. What do you think is the approximate value of the mean number of pets for the population of students at your school?

The mean number of pets is 2.

Answers will vary but should be realistic.

b. How could you find a better estimate of this population mean?

I could ask a random sample of students how many pets they have and then calculate the sample mean.

I calculate the mean by calculating the sum of all the data values and dividing by the number of data values. Due to sampling variability, I know each sample could have a different mean.

G7-M5-Lesson 18: Sampling Variability and the Effect of Sample Size

The distances, in miles, that 200 people have to travel to the airport are recorded in the table below.

	0	1	2	3	4	5	6	7	8	9
00	45	58	49	78	59	36	52	39	70	51
01	50	45	45		71	55	65	33	60	51
02	53	83	40	51	83	57	75	38	43	77
03	49	49	81	57	42	36	22	66	68	52
04	60	67	43	60	55	63	56	44	50	58
05	64	41	67	73	55	69	63	46	50	65
06	54	58	53	55	51	74	53	55	64	16
07	28	48	62	24	82	51	64	45	41	47
08	70	50	38	16	39	83	62	50	37	58
09	79	62	45	48	42	51	67	68	56	78
10	61	56	71	55	57	77	48	65	61	62
11	65	40	56	47	44	51	38	68	64	40
12	53	22	73	62	82	78	84	50	43	43
13	81	42	72	49	55	65	41	92	50	60
14	56	44	40	70	52	47	30	9	58	53
15	84	64	64	34	37	69	57	75	62	67
16	45	58	49	78	59	36	52	39	70	51
17	50	45	45	66	71	55	65	33	60	51
18	53	83	40	51	83	57	75	38	43	77
19	49	49	81	57	42	36	22	66	68	52

1. Using the random digit table, the following 15 values were chosen.

$$82, 64, 40, 64, 33, 81, 22, 50, 66, 51, 78, 49, 70, 58, 84$$

I add the data values together and divide by the number of data values.

Calculate the mean of the sample.

$$\frac{82+64+40+64+33+81+22+50+66+51+78+49+70+58+84}{15} = \frac{892}{15} \approx 59.5$$

The average distance people travel to the airport is approximately 59.5 ***miles.***

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2. Using the random digit table, the following 25 values were chosen.

$$45, 67, 71, 49, 50, 38, 67, 77, 55, 45, 64, 22, 36, 9, 49, 30, 51, 70, 75, 36, 73, 62, 79, 50, 40$$

Calculate the mean of the sample.

$$\frac{1310}{25} = 52.4$$

The average distance people travel to the airport is approximately 52.4 ***miles.***

3. Which sample mean would you expect to be closer to the population mean? Explain your reasoning.

The sample mean from Problem 2 would be closer the population mean because the sample size is greater.

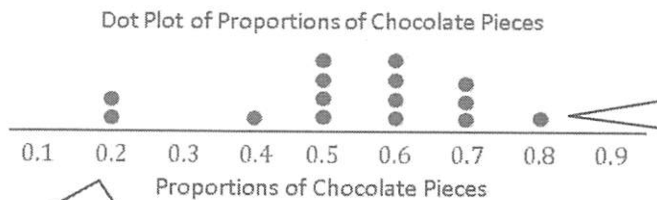
> The sample variability is smaller with a larger sample and larger with a smaller sample.

G7-M5-Lesson 19: Understanding Variability When Estimating a Population Proportion

A group of friends want to determine the number of chocolate pieces of candy in a bag of mixed candy. Each friend took a random sample of 10 pieces of candy. The table below shows the proportion of chocolate pieces of candy each friend found.

0.2	0.7	0.5	0.7	0.6
0.5	0.4	0.7	0.6	0.5
0.2	0.8	0.6	0.6	0.5

1. Construct a dot plot of the sample proportions.



Each dot represents one data value. If a value is represented more than once in the distribution, I place the additional dots above the original dot.

The number line of a dot plot must have a constant scale. I cannot skip numbers, even if a number is not represented in the distribution.

2. Describe the variability of the distribution.

The spread of the data is 0.2 *to* 0.8*; however, most of the data is between* 0.5 *and* 0.7*.*

The spread explains the location of the data values using the minimum and maximum.

I know the sampling variability decreases when my sample size increases.

3. Suppose each friend picked 40 pieces of candy from the bag. Describe how the sampling distribution would change from the one you constructed in Problem 1.

The sampling variability would decrease.

G7-M5-Lesson 20: Estimating a Population Proportion

A group of 20 seventh graders wanted to estimate the proportion of middle school students who buy school lunch every day. Each seventh grader took a random sample of 20 middle school students and asked each student whether or not he or she bought lunch. Following are the sample proportions the seventh graders found in 20 samples.

0.15	0.10	0.20	0.00	0.05
0.25	0.30	0.00	0.10	0.15
0.10	0.05	0.20	0.10	0.10
0.15	0.15	0.20	0.00	0.10

I can write this number as a fraction, $\frac{25}{100}$. To determine how many students in the sample buy school lunch every day, I find an equivalent fraction where the denominator is the sample size.

1. One of the seventh graders reported a sample proportion of 0.25. What does this value mean in terms of the scenario?

   $\frac{25}{100} = \frac{5}{20}$

   A sample proportion of 0.25 means that 5 out of 20 students in the sample buy school lunch every day.

2. Construct a dot plot of the 20 sample proportions.

3. Describe the shape of the distribution.

The shape of the distribution is symmetric.
It centers at approximately 0.10.

A majority of the data values center around the same number. I notice the dot plot looks like a mound, so it would have a symmetric shape.

4. Using the 20 sample proportions listed above, what is your estimate for the proportion of all the middle school students who bought school lunch every day?

My estimate for the proportion of all middle school students who bought school lunch every day is 0.12 *because I think the proportion will be between* 0.10 *and* 0.15.

Answers will vary but should be close to the actual mean of 0.1225.

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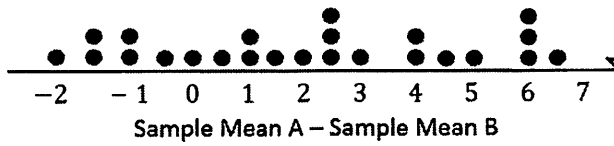
G7-M5-Lesson 21: Why Worry About Sampling Variability?

Below are two dot plots. Each dot plot represents the differences in sample means for random samples selected from two populations (Bag A and Bag B). For each distribution, the differences were found by subtracting the sample means of Bag B from the sample means of Bag A (sample mean A – sample mean B).

1. Examine the dot plot below.



I notice that a majority of the differences are a positive value.

a. Does the dot plot above indicate that the population mean of Bag A is larger than the population mean of Bag B? Why or why not?

The population mean of Bag A is larger than the population mean of Bag B because a majority of the differences are positive, which means the sample means for Bag A were larger than the sample means for Bag B.

Due to the order of the subtraction, if the population mean of Bag B were bigger than Bag A, a majority of the differences would be negative.

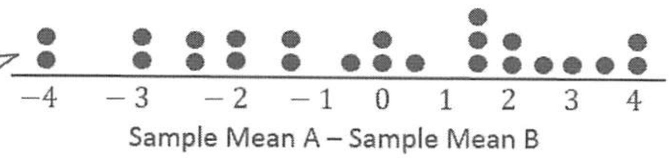
b. In the above graph, how many differences are greater than 0? How many differences are less than 0? What might this tell you?

There are 17 differences that are greater than 0 and only 6 differences that are less than 0. This would tell me that the population mean for Bag A is most likely larger than the population mean for Bag B because a larger number minus a smaller number results in a positive number.

It is possible that the population mean of Bag B is larger than the population mean of Bag A, but it is not very likely since there are a lot more positive differences than negative differences.

2. Examine the dot plot below.

The dot plot is centered around 0, so I expect that the difference of the two population means is also close to 0.



Does the dot plot above indicate that the population mean of Bag A is larger than the population mean of Bag B? Why or why not?

The dot plot indicates that the population means of both bags are about the same because there are the same number of positive and negative values on the dot plot.

G7-M5-Lesson 22: Using Sample Data to Compare the Means of Two or More Populations

Measure of Variability

1. A school is trying to decide which math program to purchase.

   a. How many mean absolute deviations (MADs) separate the mean mathematics score for the Math Facts program (mean $= 42.7$, MAD $= 3.7$, $n = 32$) and the Math Genius program (mean $= 38.6$, MAD $= 4.0$, $n = 28$)?

$$\frac{47.7 - 38.6}{4.0} = 2.275$$

I subtract the two sample means and then divide by the MAD. If the two MADs are different, I use the larger of the two MADs.

This value indicates that the data are separated by a little more than 2 MADs.

The number of MADs that separate the sample mean mathematics score for the Math Facts program and the Math Genius program is 2.275*, a little more than two MADs.*

   b. What recommendation would you make based on the result?

The number of MADs that separate the two programs is significant, so I would recommend the Math Facts program because it produces higher scores.

I know Math Facts produces significantly higher scores because the mean is higher than the Math Genius mean.

In general, if the MADs are separated by 2 or more, then this is significant.

2. Does a pickup truck or an SUV get better gas mileage? A sample of 10 different cars and pickup trucks and their gas mileage (miles per gallon) is provided in the table below.

Trucks	16	15	19	18	18	21	17	19	20	20
SUVs	20	20	20	23	25	23	24	22	30	26

a. Calculate the difference between the sample mean gas mileage for the trucks and for SUVs.

Sample mean gas mileage (in miles per gallon) for trucks:

$$\frac{16 + 15 + 19 + 18 + 18 + 21 + 17 + 19 + 20 + 20}{10} = 18.3$$

To calculate the mean, I add my data values together and divide by the number of data values.

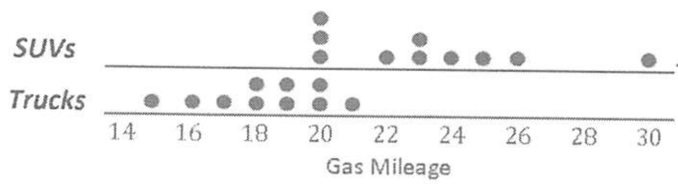
Sample mean gas mileage (in miles per gallon) for SUVs:

$$\frac{20 + 20 + 20 + 23 + 25 + 23 + 24 + 22 + 30 + 26}{10} = 23.3$$

$23.3 - 18.3 = 5$

The difference between the two sample means is 5.

b. On the same scale, draw dot plots of the two distributions, and discuss the variability in each distribution.



The dots on the SUVs dot plot are a little more spread out than the dots on the trucks dot plot.

The SUVs have a little larger variability than the trucks.

To calculate the MAD, I first need to determine the deviations or the distance each point is from the mean.

c. Calculate the MAD for each distribution. Based on the MADs, compare the variability in each distribution. Is the variability about the same? Interpret the MADs in the context of the problem.

Deviations of Trucks:

$18.3 - 16 = 2.3$

$18.3 - 15 = 3.3$

$19 - 18.3 = 0.7$

$18.3 - 18 = 0.3$

$18.3 - 18 = 0.3$

$21 - 18.3 = 2.7$

$18.3 - 17 = 1.3$

$19 - 18.3 = 0.7$

$20 - 18.3 = 1.7$

$20 - 18.3 = 1.7$

Now that I know the deviations, I find the sum of the deviations and divide the sum by the number of data values.

$$\frac{2.3 + 3.3 + 0.7 + 0.3 + 0.3 + 2.7 + 1.3 + 0.7 + 1.7 + 1.7}{10} = \frac{15}{10} = 1.5$$

I follow the same process to calculate the MAD for SUVs.

SUVs:

$$\frac{3.3 + 3.3 + 3.3 + 0.3 + 1.7 + 0.3 + 0.7 + 1.3 + 6.7 + 2.7}{10} = \frac{23.6}{10} = 2.36$$

The MAD for trucks is 1.5*, which means the typical deviation from the mean of* 18.3 *is* 1.5*.*

The MAD for SUVs is 2.36*, which means the typical deviation from the mean of* 23.3 *is* 2.36*.*

d. Based on your calculations, is the difference in mean distance meaningful?

$$\frac{5}{2.36} = 2.11$$

There is a separation of 2.11 *MADs. There is a meaningful distance between the means.*

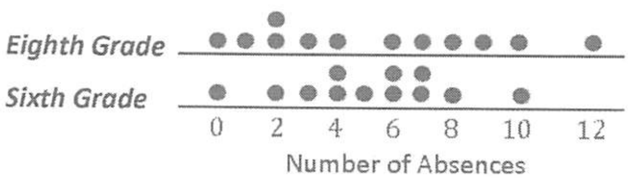
I know a meaningful distance is similar to a significant difference.

G7-M5-Lesson 23: Using Sample Data to Compare the Means of Two or More Populations

Principal MacDonald wanted to determine if sixth graders or eighth graders were absent more often. The table below shows the number of absences 12 students in each grade had throughout the year.

Sixth Grade	4	0	6	10	8	7	3	2	4	7	6	5
Eighth Grade	0	8	12	4	7	6	9	10	1	3	2	2

1. On the same scale, draw dot plots for the two sample data sets.



2. Looking at the dot plots, list some observations comparing the number of absences for sixth graders and the number of absences for eighth graders.

The dot plots look similar. The variability is slightly larger for eighth grade students than for sixth grade students. However, both data distributions look like they have means close to each other.

Although answers may vary a little, it is important for students to notice the dot plots are similar.

An additional explanation for how to calculate this information can be found in Lesson 22.

3. Calculate the mean and MAD for each of the data sets. Round to the nearest hundredth if necessary.

	Mean (days)	MAD (days)
Sixth Grade	5.17	2.17
Eighth Grade	5.33	3.33

I add all the data values and divide by the number of data values.

I remember from Lesson 22 that I calculate the sum of the deviations and divide by the number of data values.

4. How many MADs separate the two sample means?

$$\frac{5.33 - 5.17}{3.33} \approx 0.05$$

I calculate the difference between the means and divide by the largest MAD.

5. What can you say about the average number of absences for all sixth graders in the population compared to the average number of absences for all eighth graders in the population?

Since the number of MADs that separate the means is approximately 0.05***, we can assume sixth graders and eighth graders miss about the same amount of school.***

In order to conclude that students in one grade miss more school than another grade, the separation between the means needs to be significant.